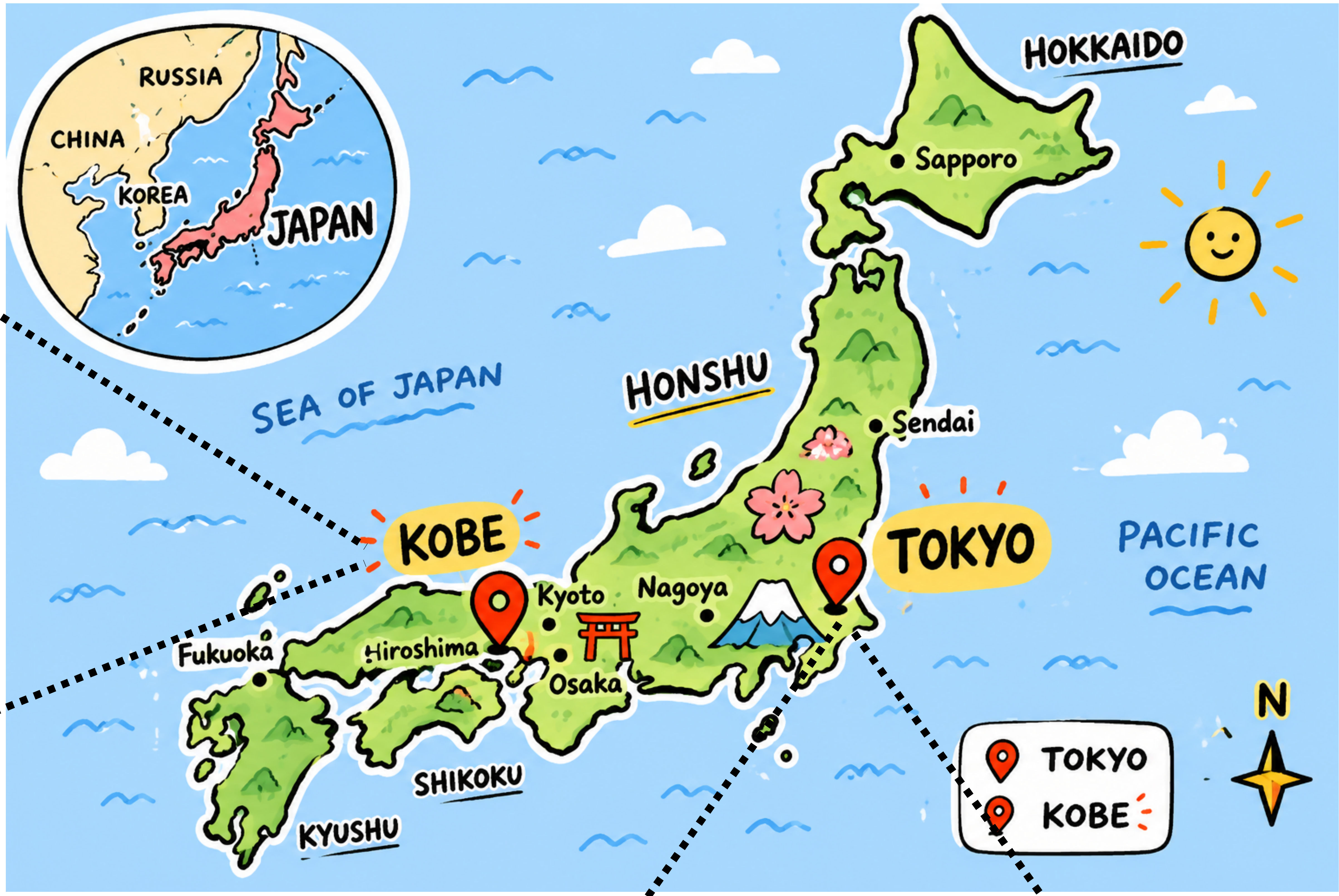
Direct reciprocity under constraints: information & evolutionary dynamics

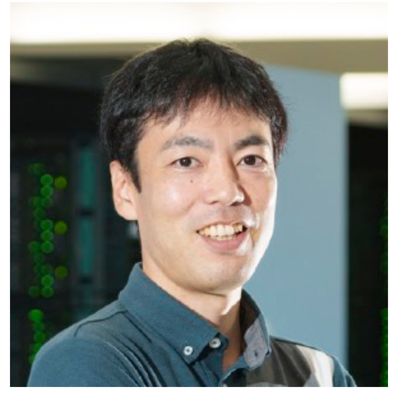
Game theory in ecology and evolution

Nikoleta E. Glynatsi

RIKEN R-CCS

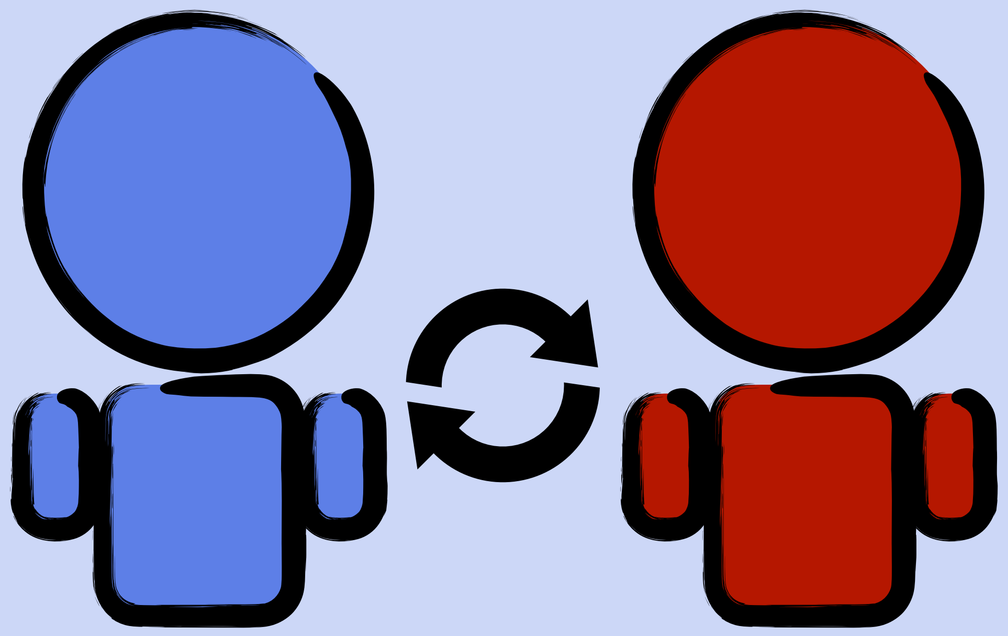


iTHEMS



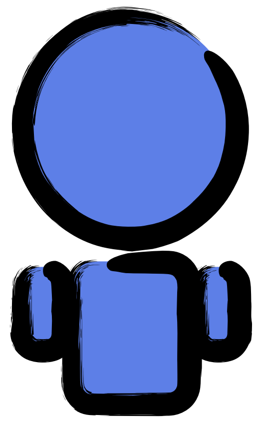
https://ithems-math-soc.github.io/

Evolutionary models of direct reciprocity

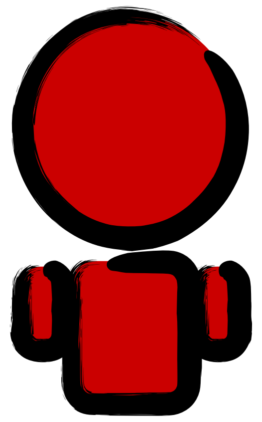


Direct reciprocity

$$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \cdots \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$$

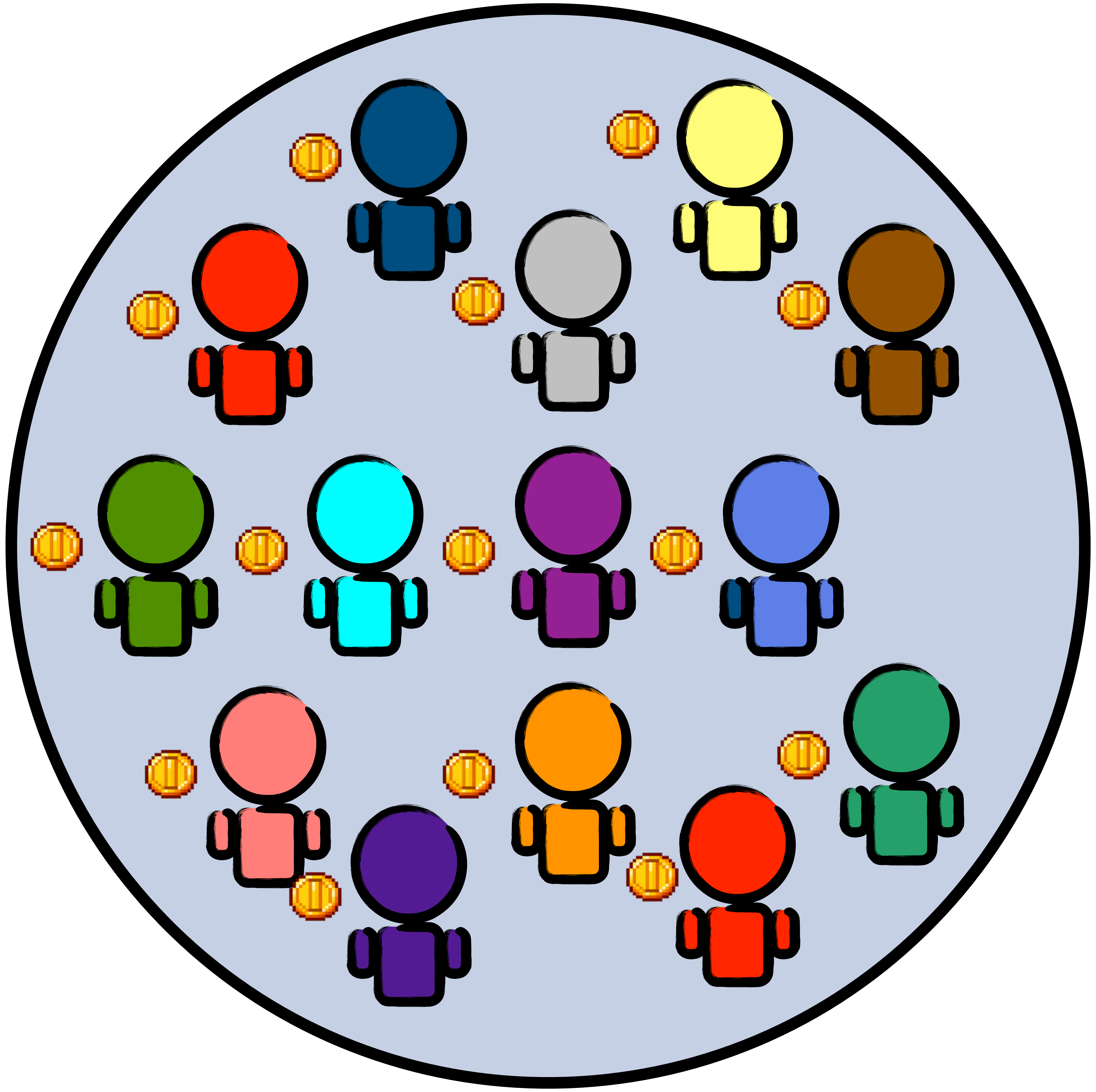


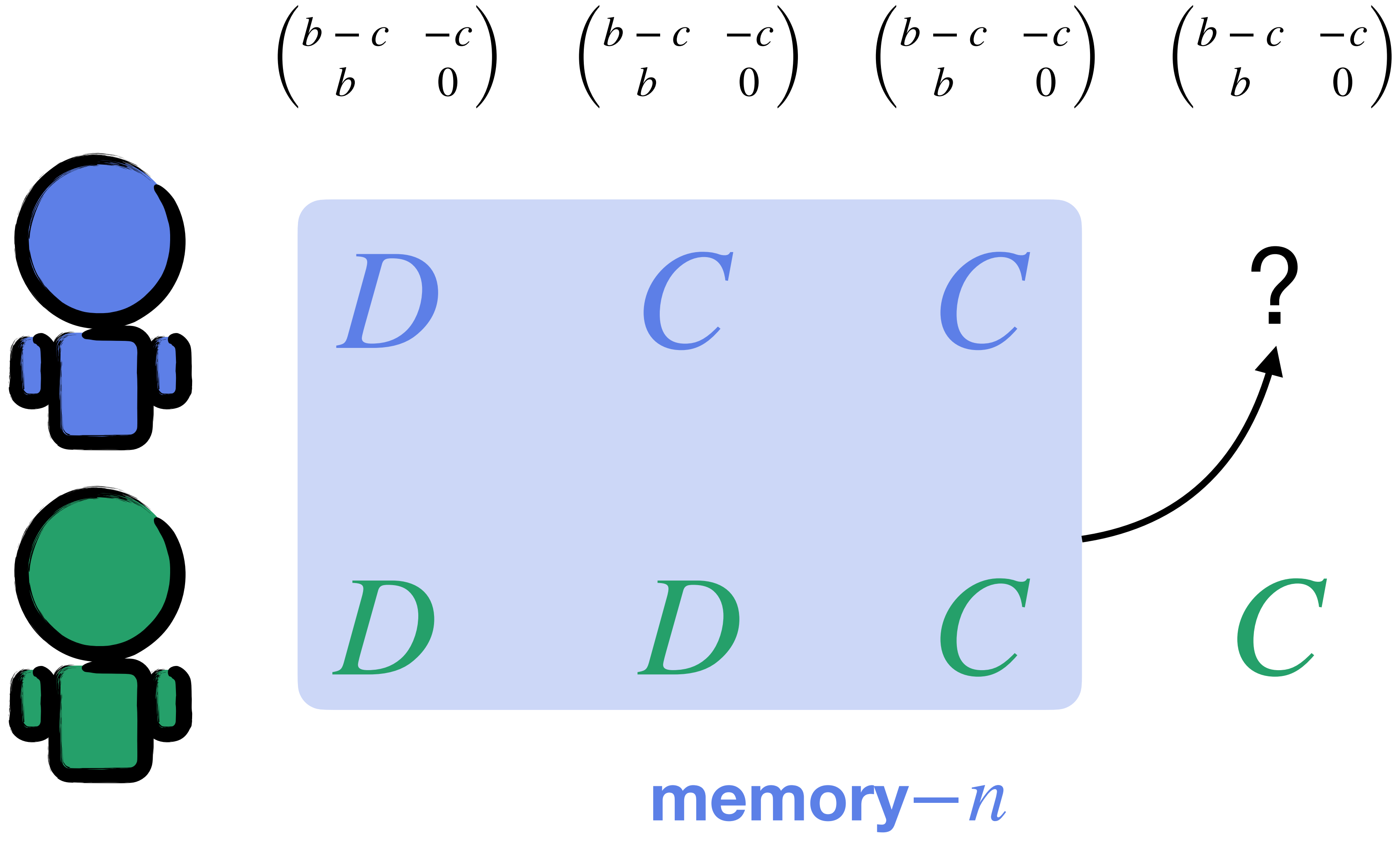
$D \quad C \quad C \quad \cdots \quad C$



$D \quad D \quad C \quad \cdots \quad C$

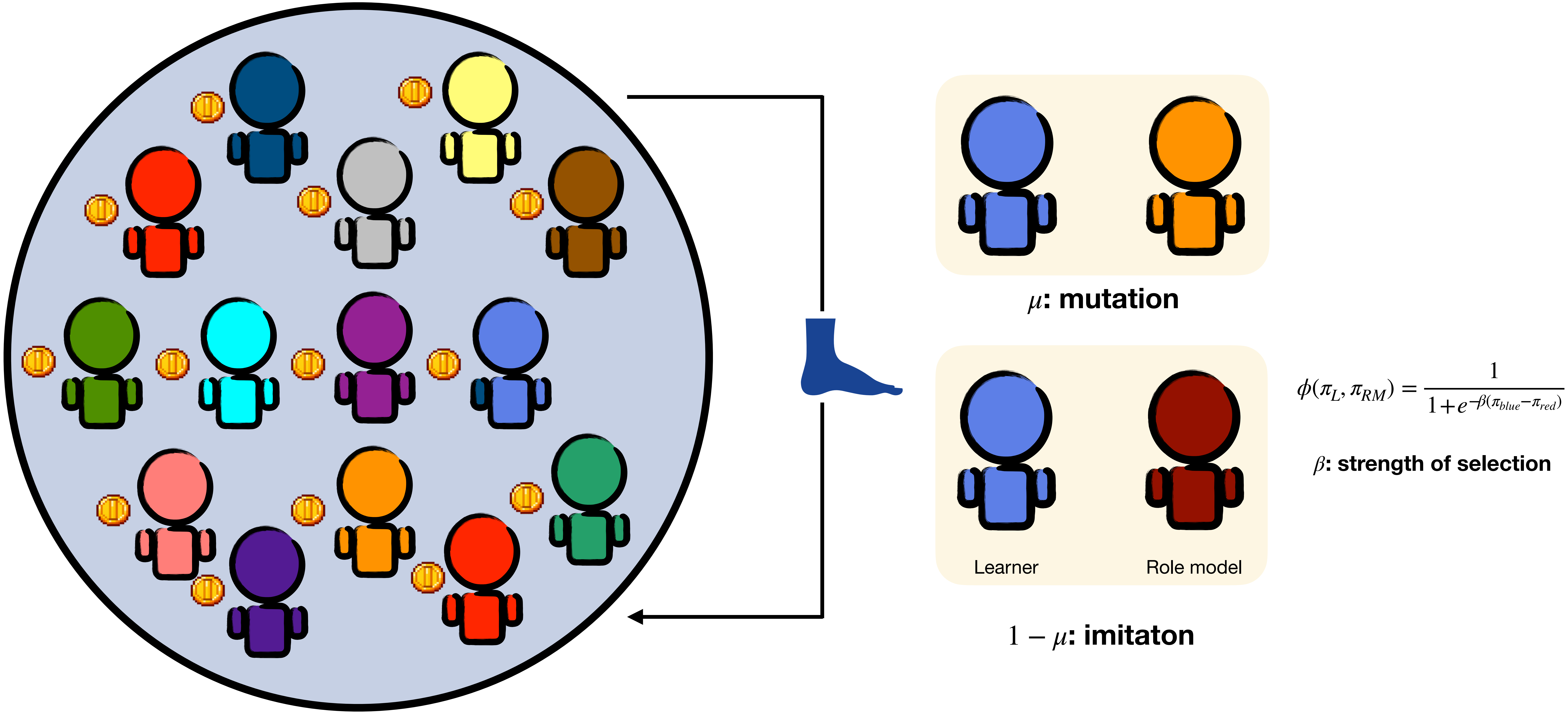
Evolutionary models of direct reciprocity



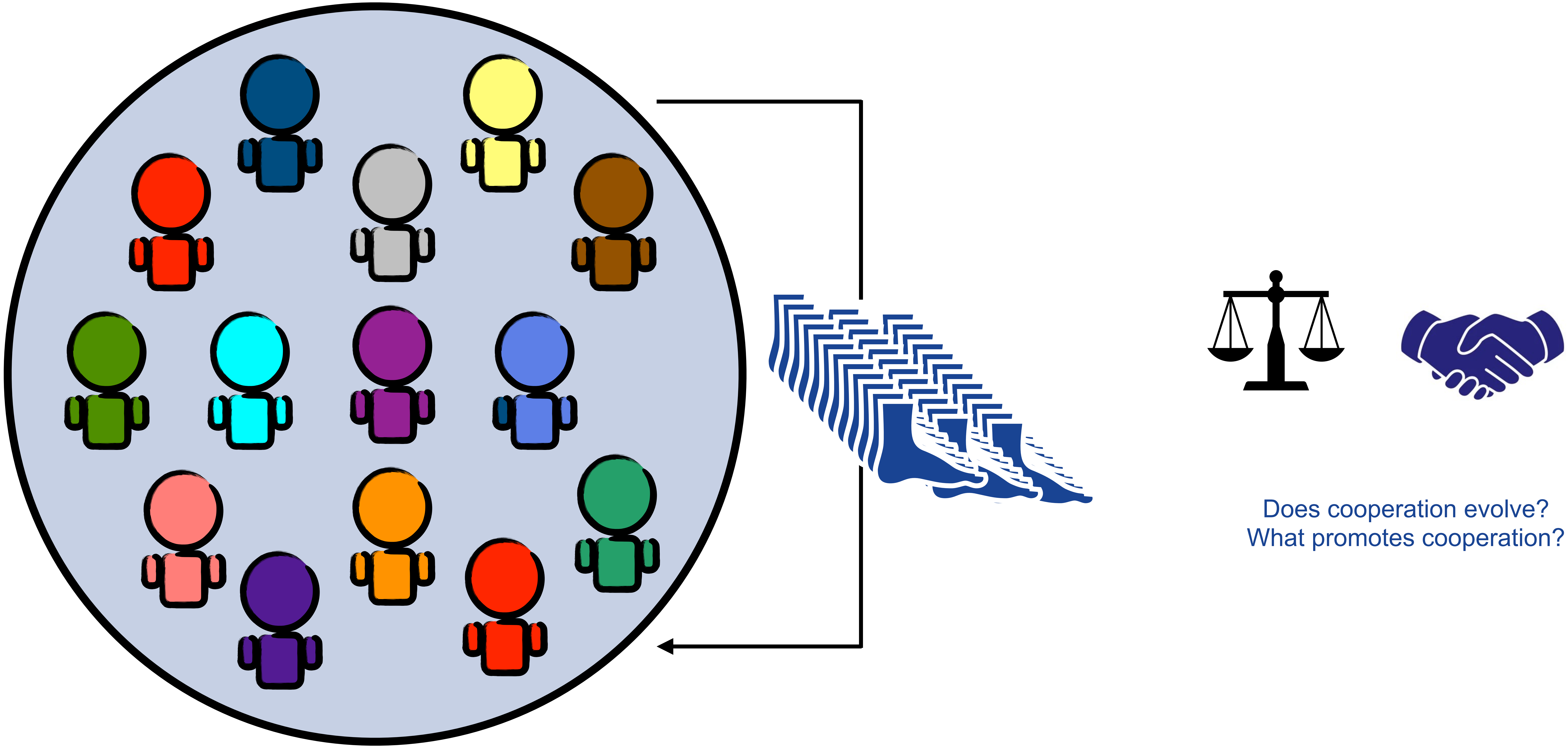


Strategies: determine behavior from observations and past interactions.

Evolutionary models of direct reciprocity

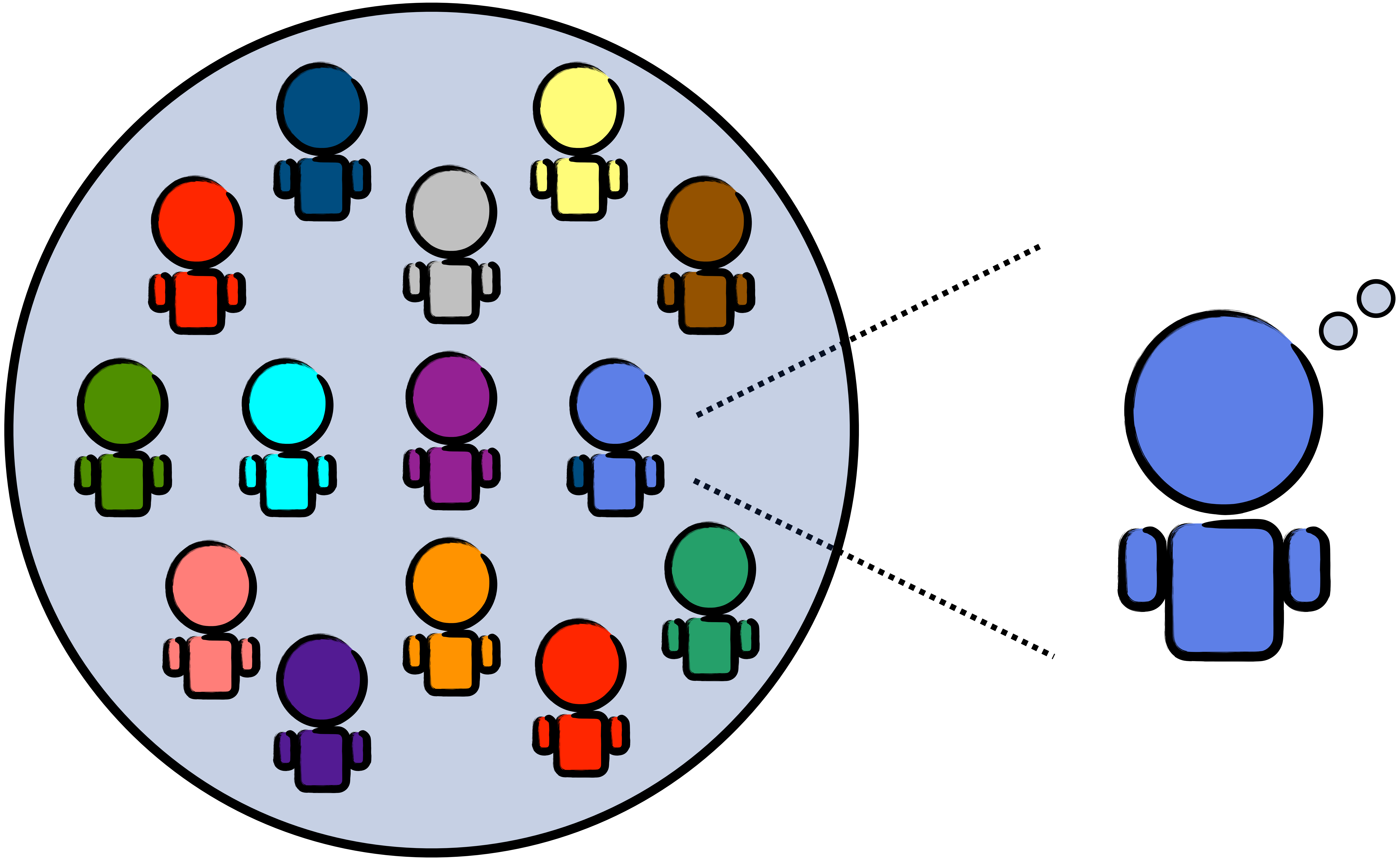


Evolutionary models of direct reciprocity



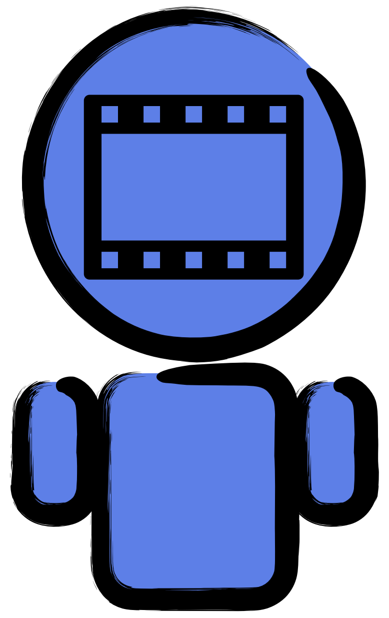
Does cooperation evolve?
What promotes cooperation?

Evolutionary models of direct reciprocity

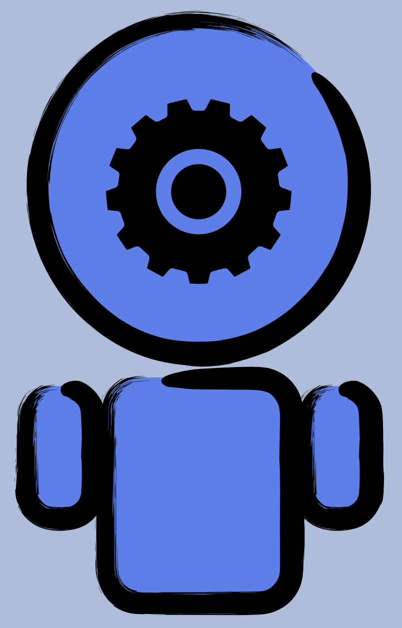


- How much **Blue** can remember
- How much information **Blue** has
- How **Blue** learns

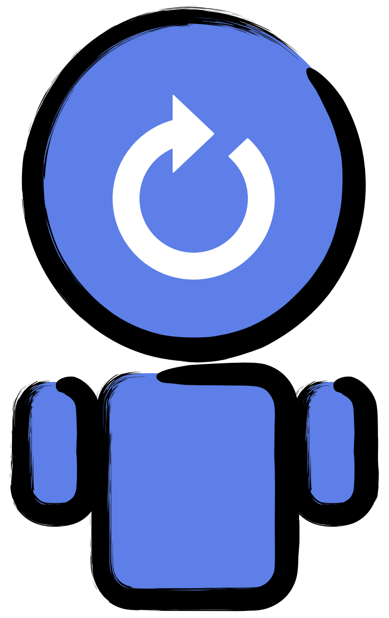
How much Blue can remember



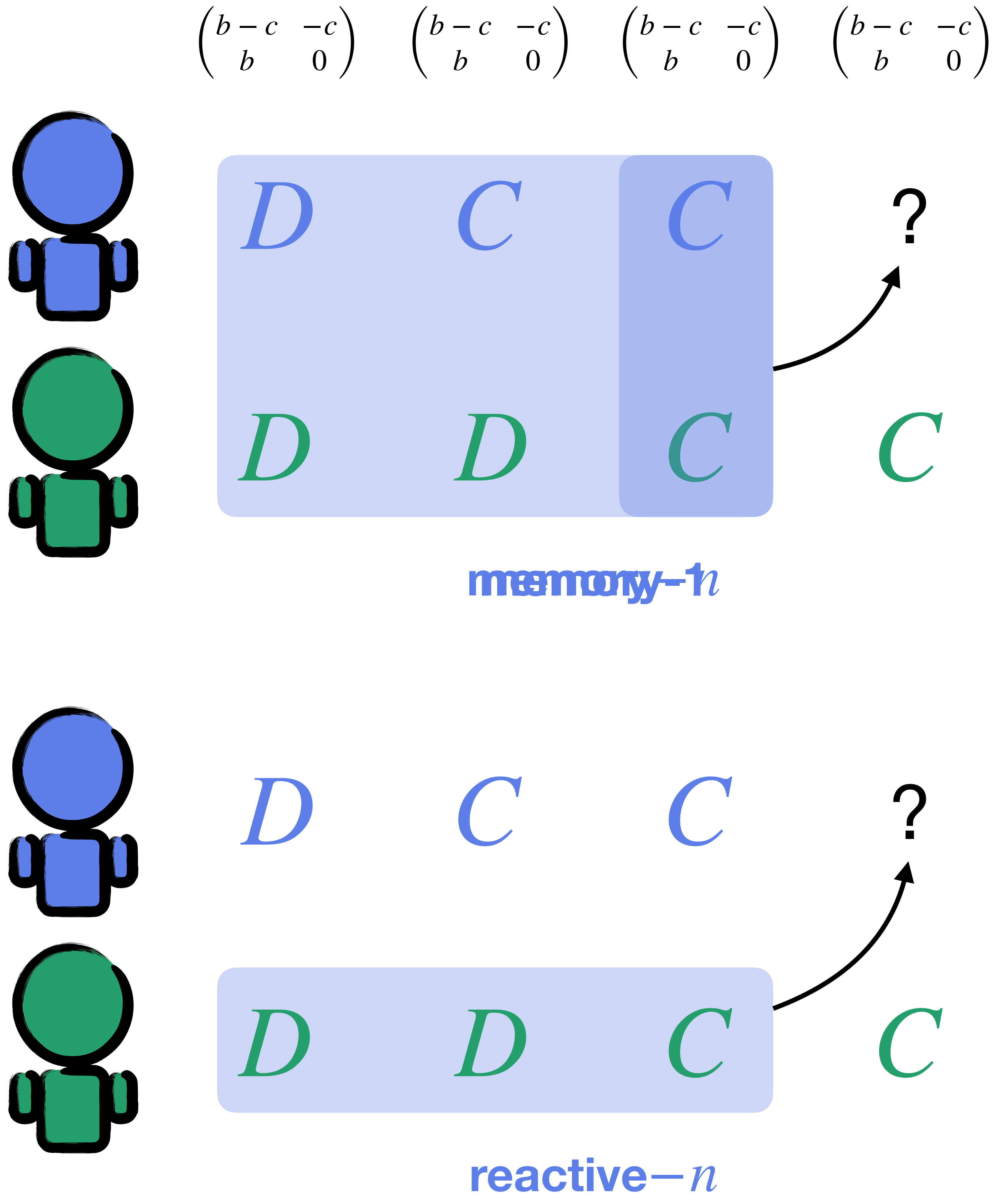
How much information Blue has

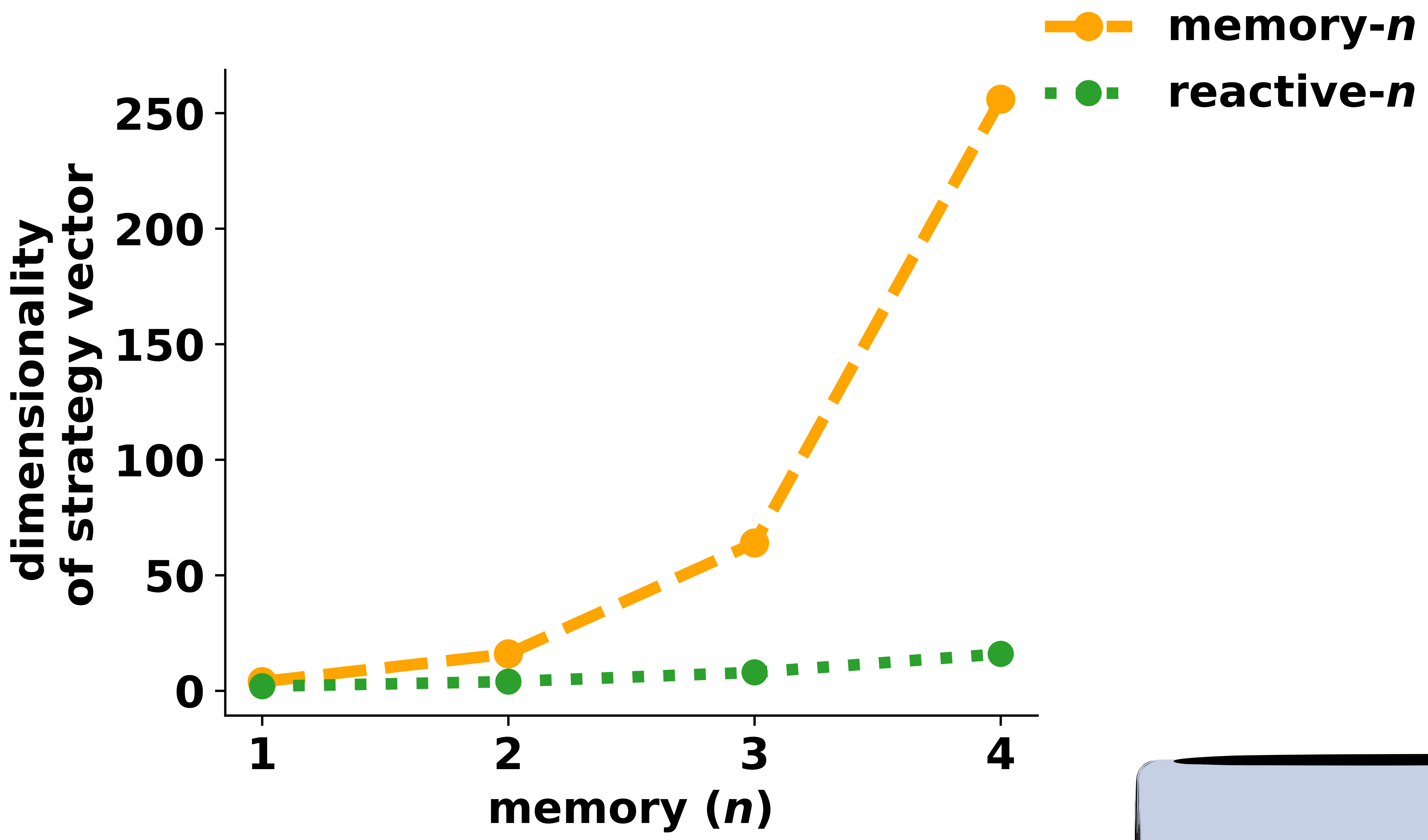


How Blue learns



Conditional cooperation with longer memory



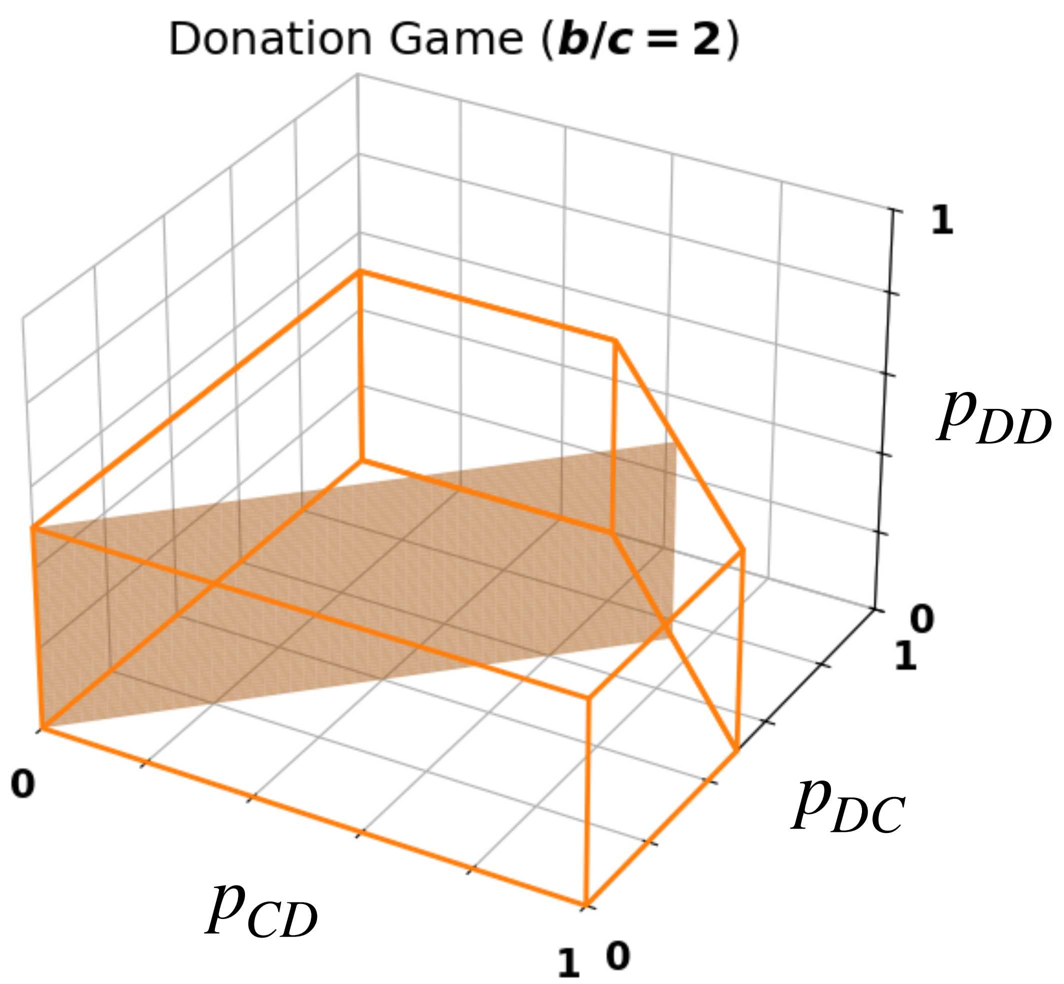


Algorithm to verify whether a given reactive-n strategy is an equilibrium

Conditional cooperation with longer memory

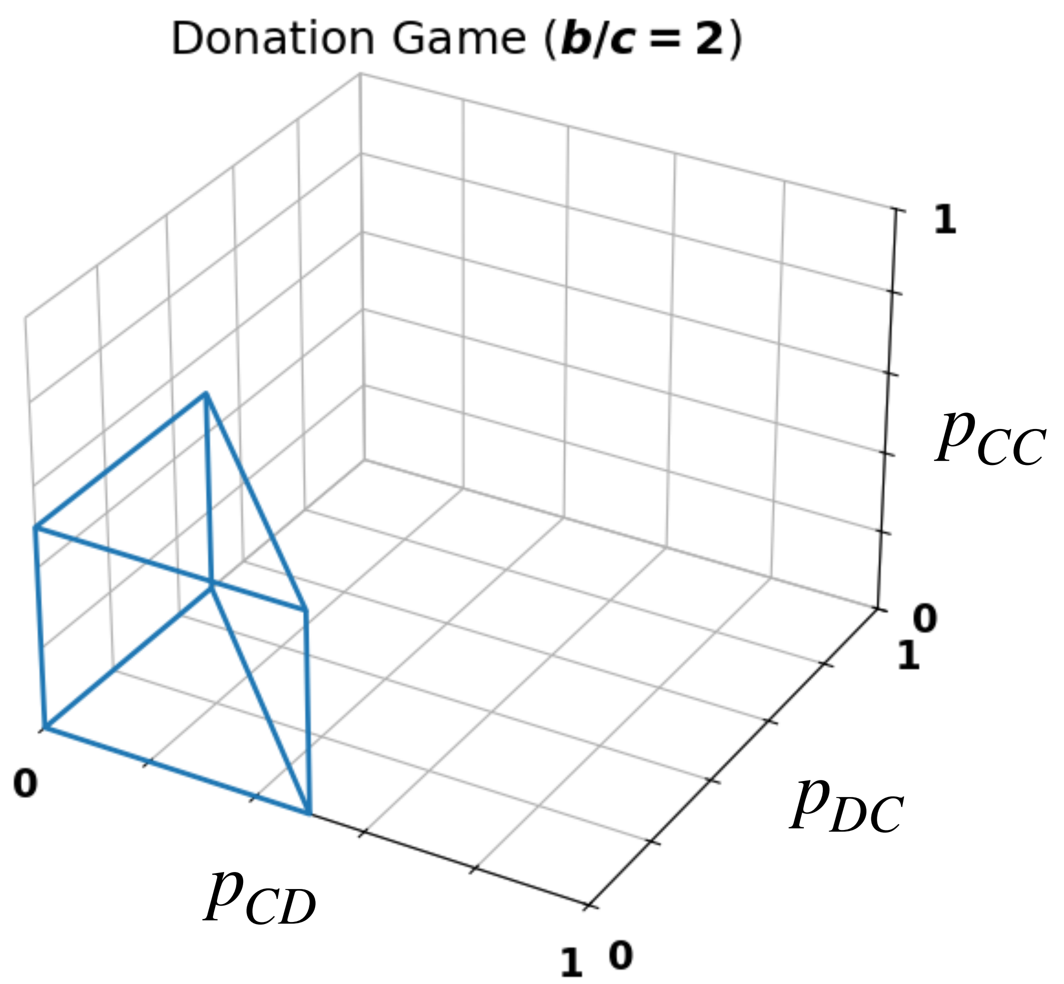
Theorem. A reactive-2 strategy $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ is a cooperative Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CC} = 1, \quad \frac{p_{CD} + p_{DC}}{2} < 1 - \frac{1}{2} \cdot \frac{c}{b}, \quad p_{DD} \leq 1 - \frac{c}{b}.$$



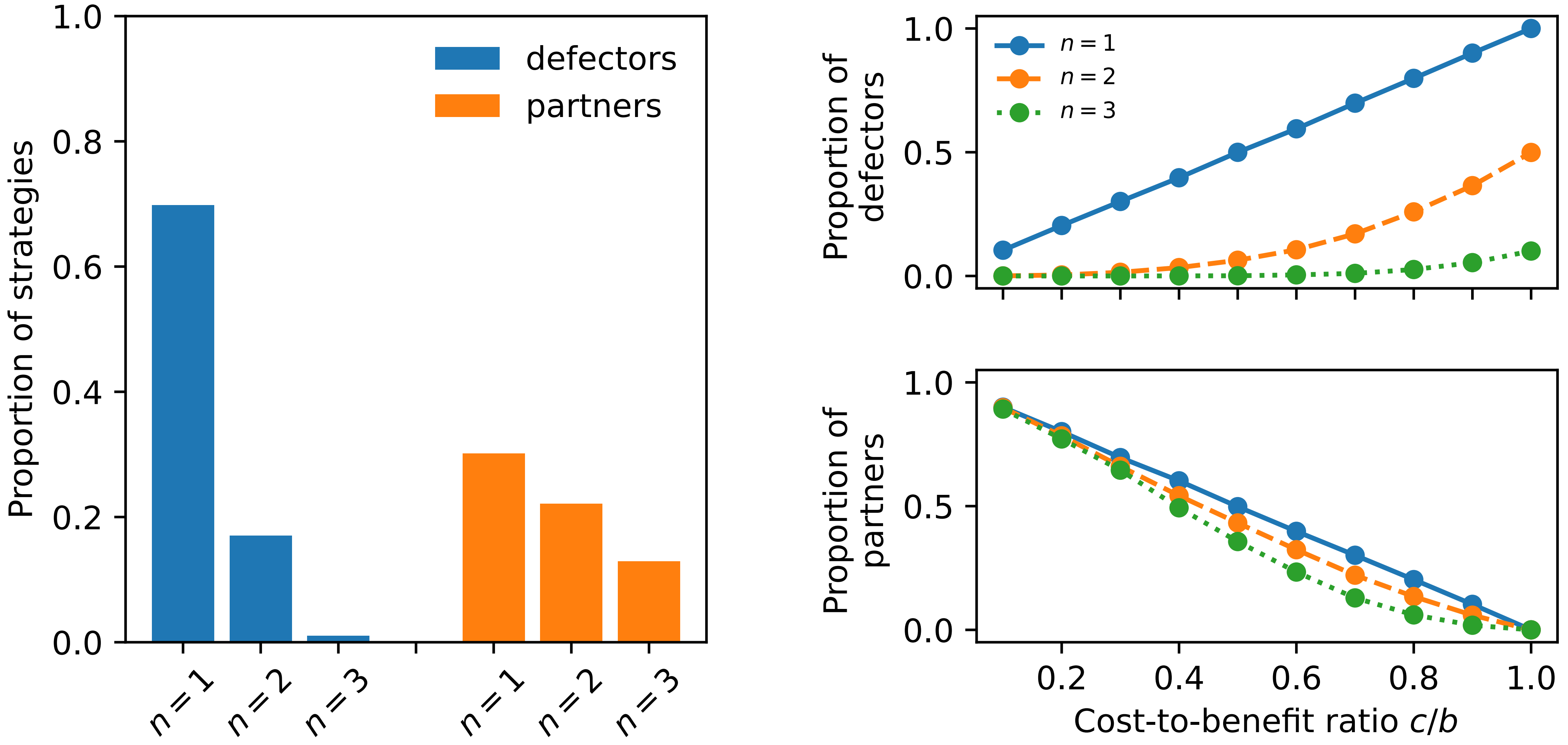
Theorem. A reactive-2 strategy $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ is a defective Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CC} \leq \frac{c}{b} \quad \frac{p_{CD} + p_{DC}}{2} \leq \frac{c}{2b}, \quad p_{DD} = 0.$$

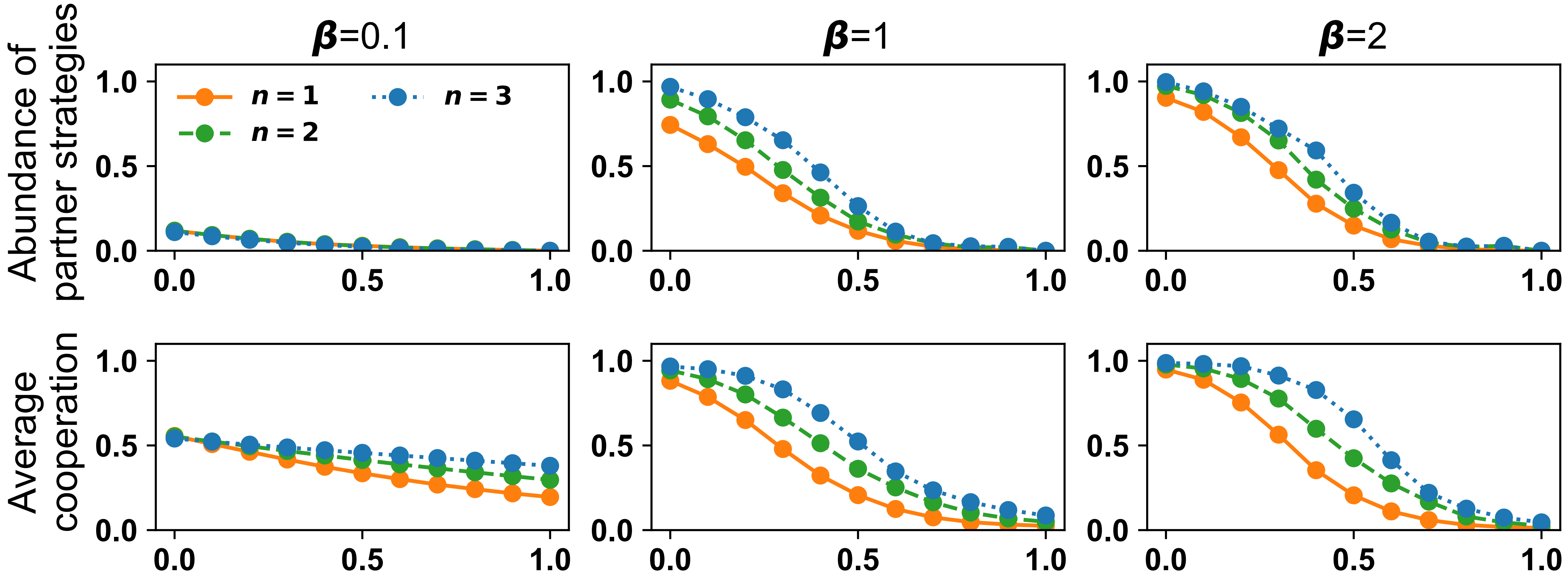


Conditional cooperation with longer memory

Volume of these Nash classes



Effect on evolutionary simulations



* Stewart, A.J. and Plotkin, J.B., 2016. Small groups and long memories promote cooperation. *Scientific reports*, *6*(1), p.26889.

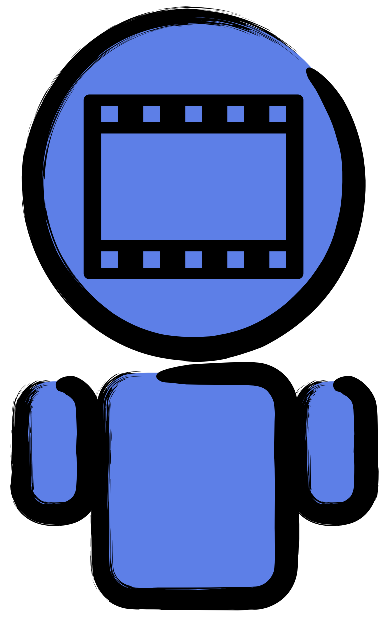
Conditional cooperation with longer memory

- Algorithm to verify whether a given reactive-n strategy is an equilibrium.
- Fully characterize cooperative & defective equilibria for $n = 2$ and $n = 3$.
- We have these classes and we can understand the effect on the evolutionary dynamics
- More memory allows for more cooperative than defective Nash and allows for more cooperation

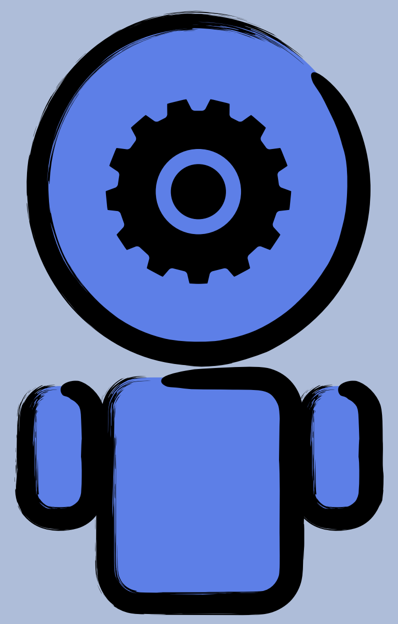
- We have this elegant algorithm. Can we characterize all Nash equilibria in the space of reactive-n strategies?
- What other classes are there?
- Which Nash equilibria become important in evolutionary simulations?
- Why do they — or do they not — become important?

Franziska Lesigang

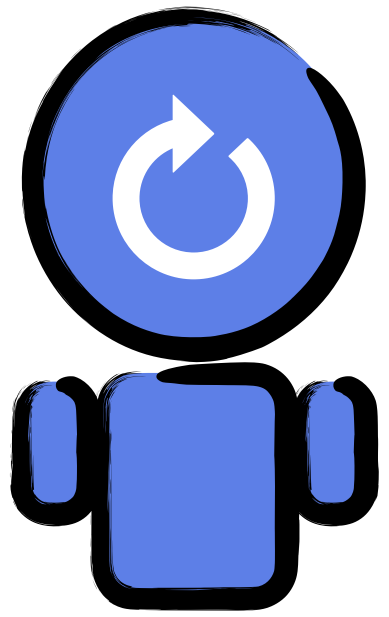
How much Blue can remember



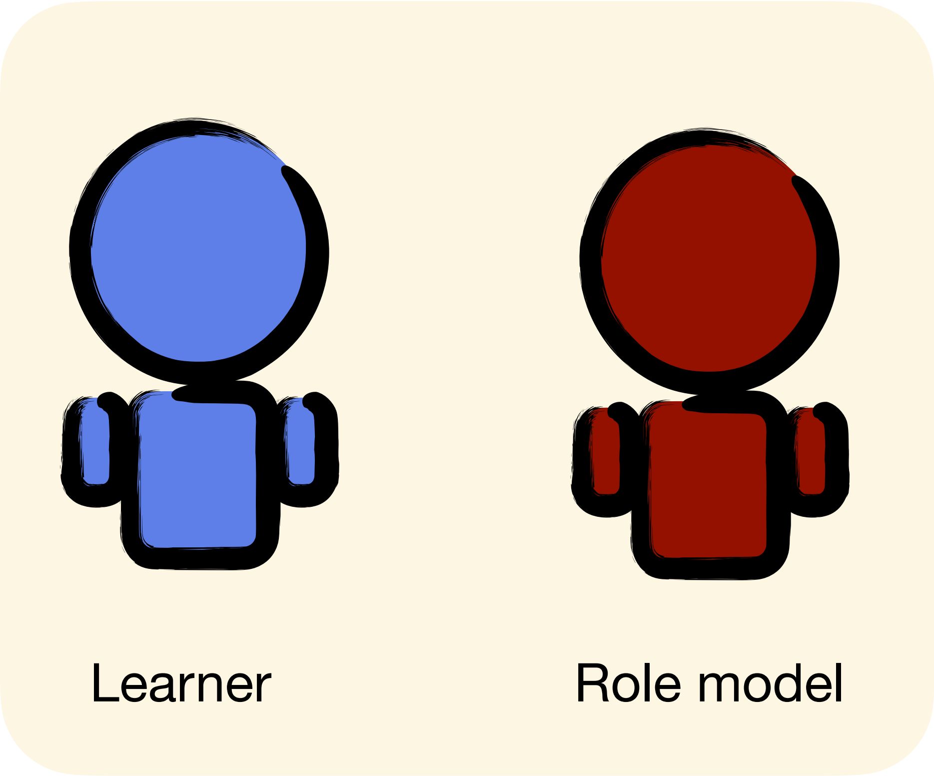
How much information Blue has



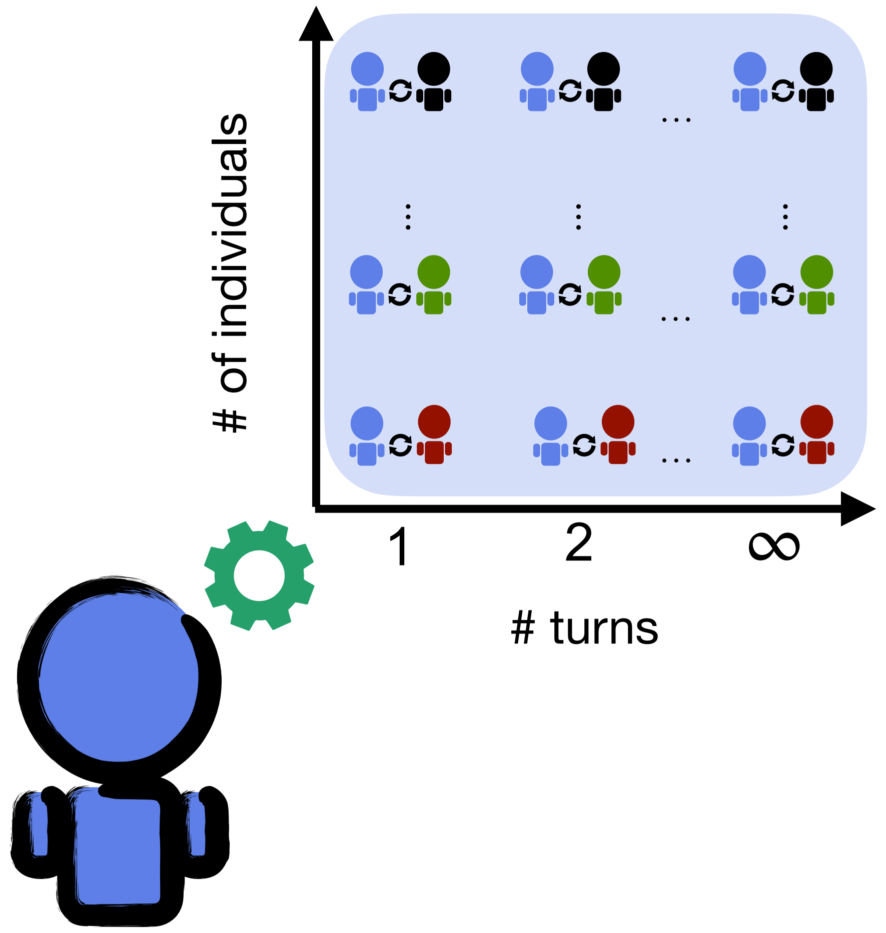
How does Blue learn

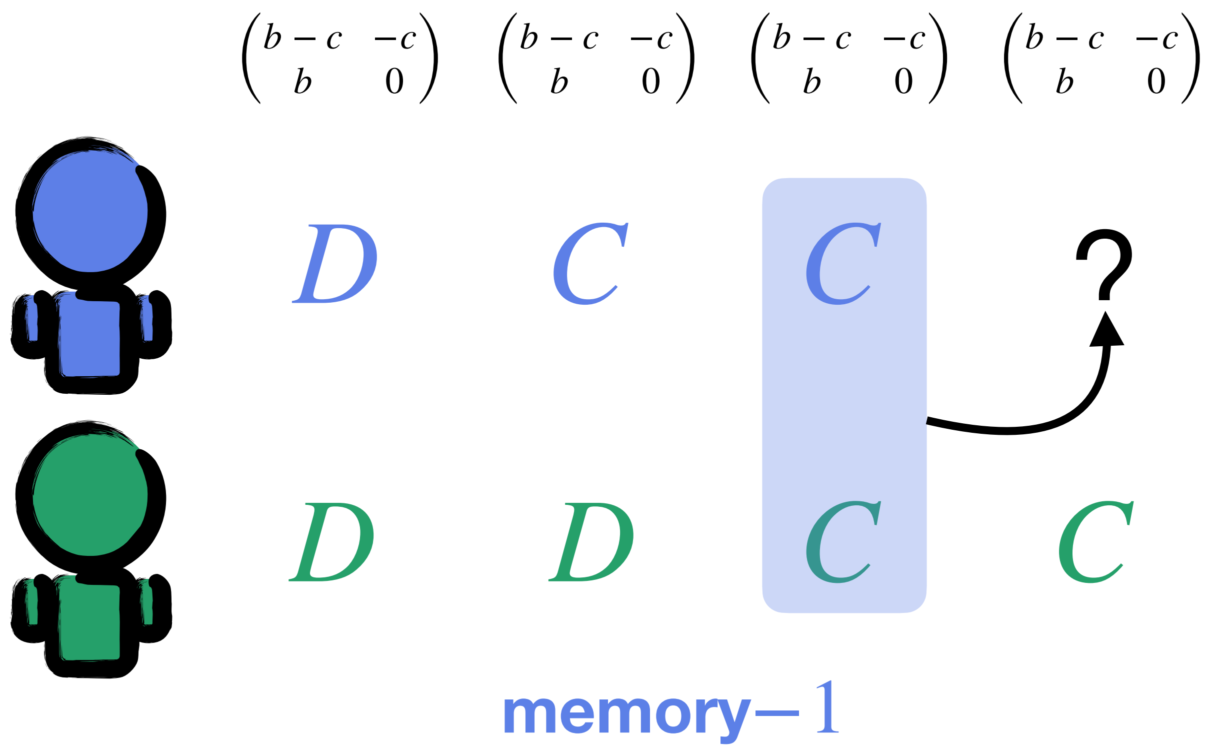


Evolution of reciprocity with limited payoff memory



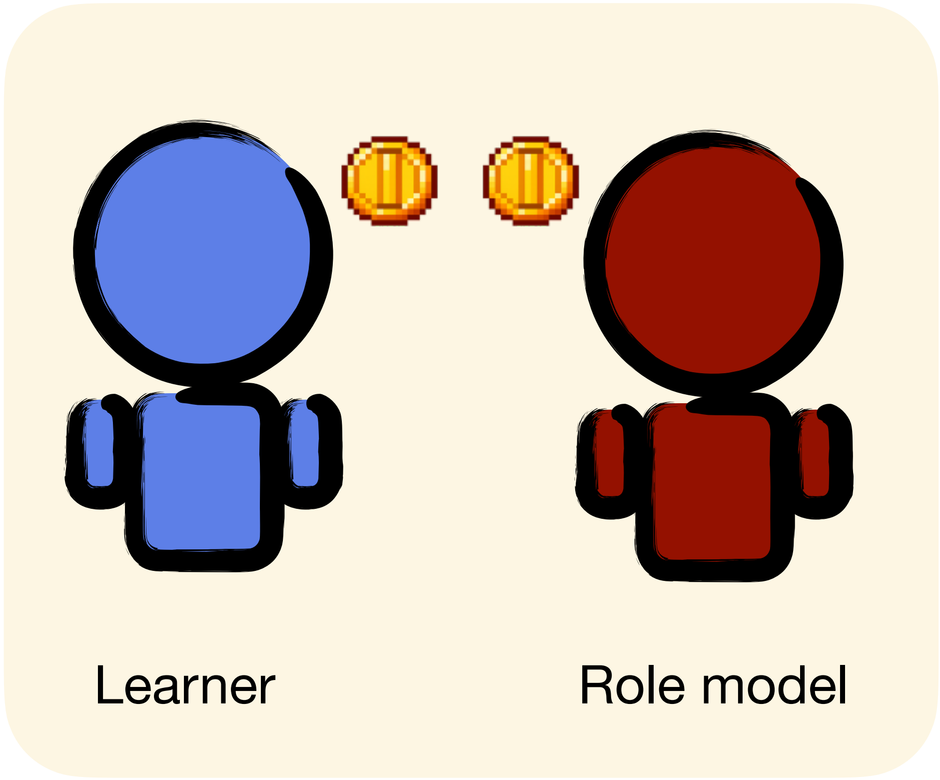
$1 - \mu$: **imitaton**





***Contradicting assumptions**

Evolution of reciprocity with limited payoff memory



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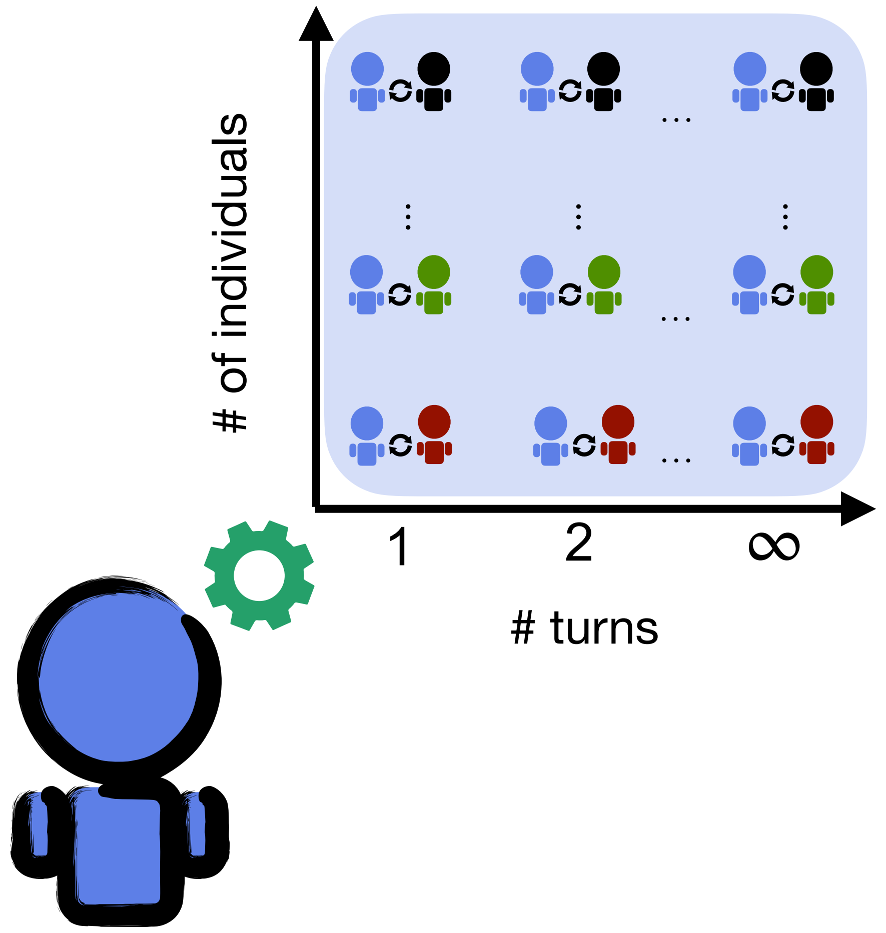
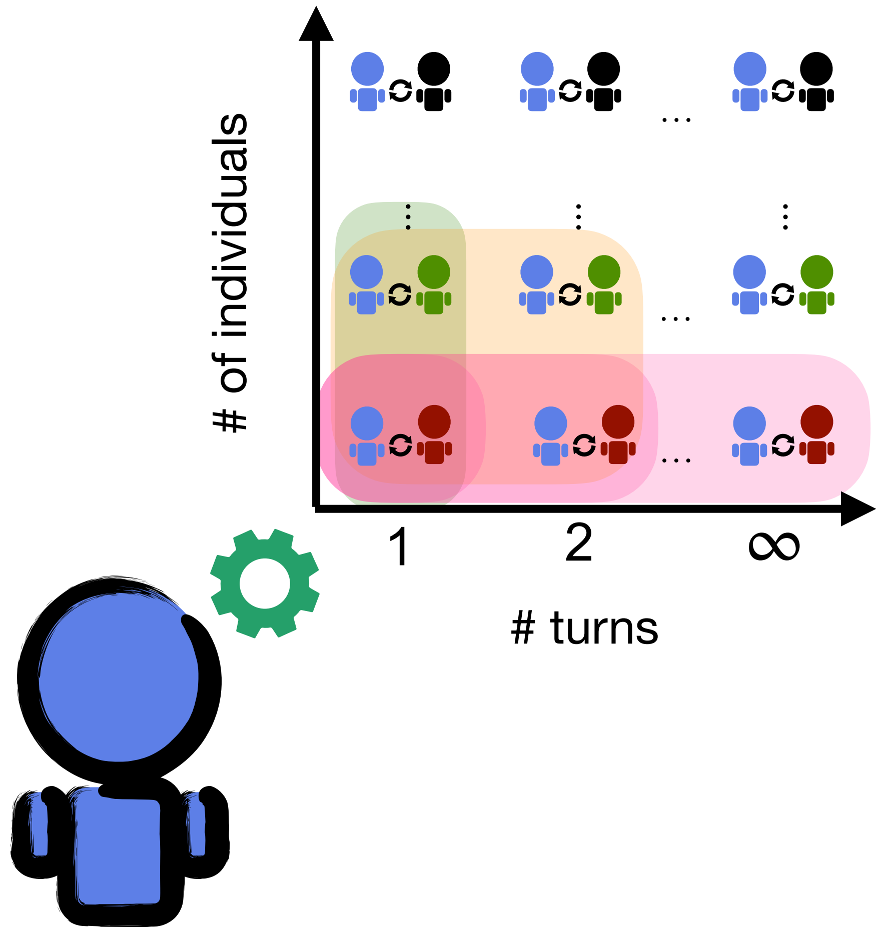
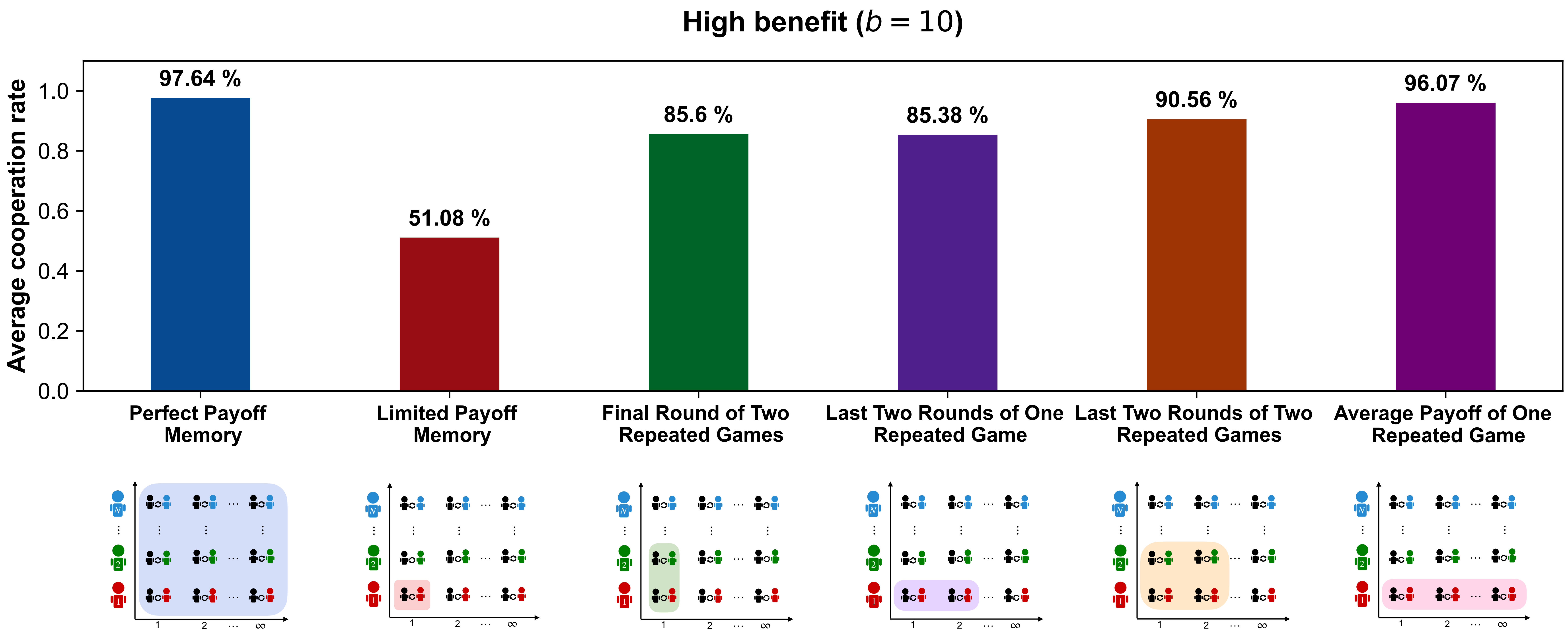




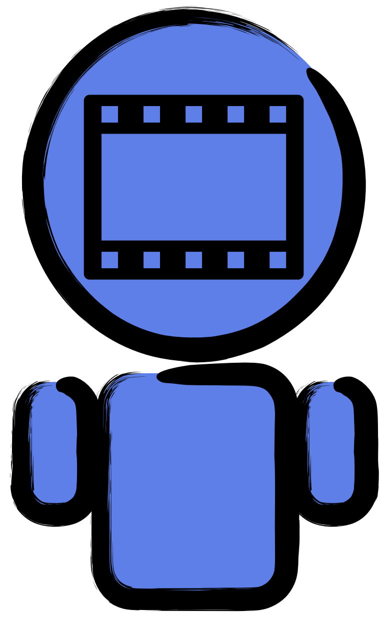
Evolution of reciprocity with limited payoff memory



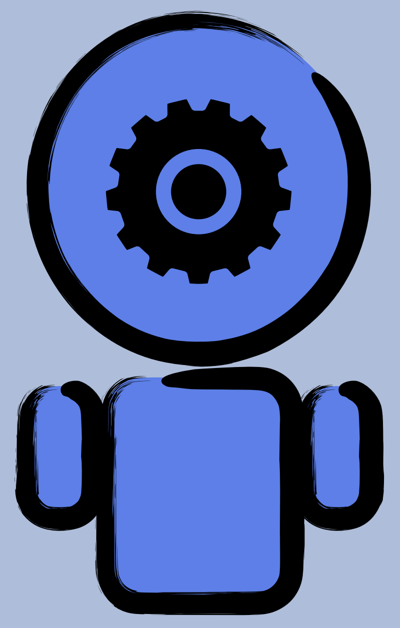
Evolution of reciprocity with limited payoff memory

- Cooperation still evolves even with limited memory
- As individuals remember two or three recent interactions, the cooperation rates approach the classical limit

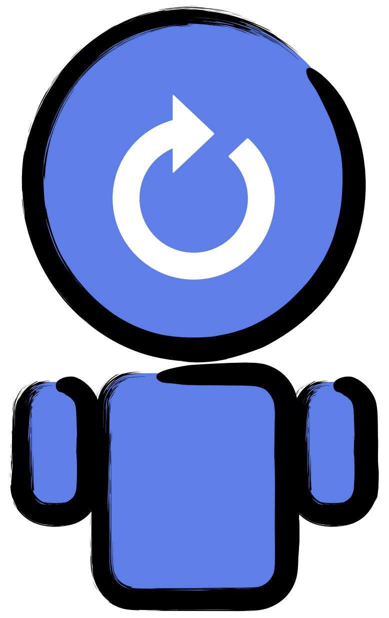
How much Blue can remember



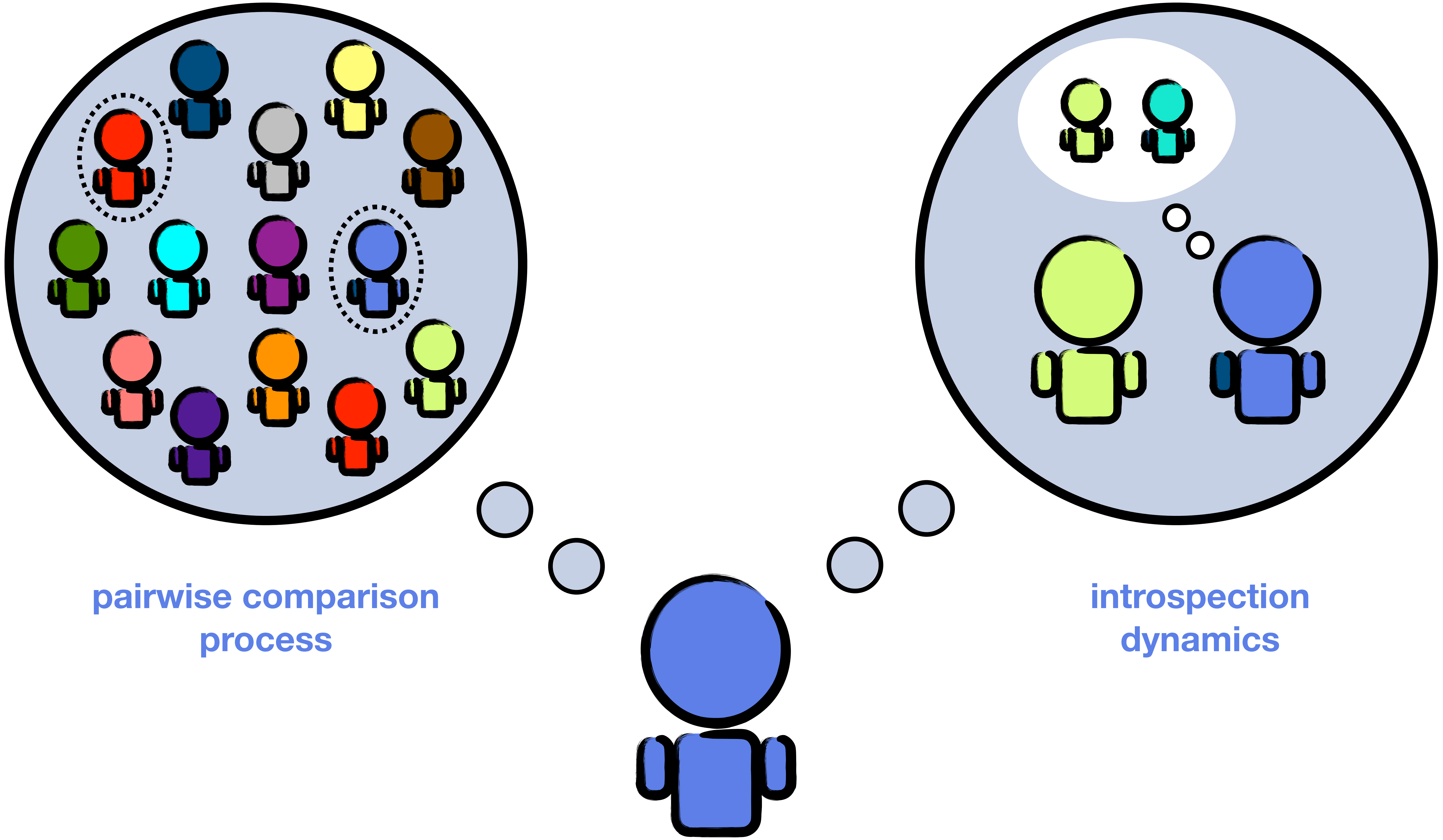
How much information Blue has



How does Blue learn

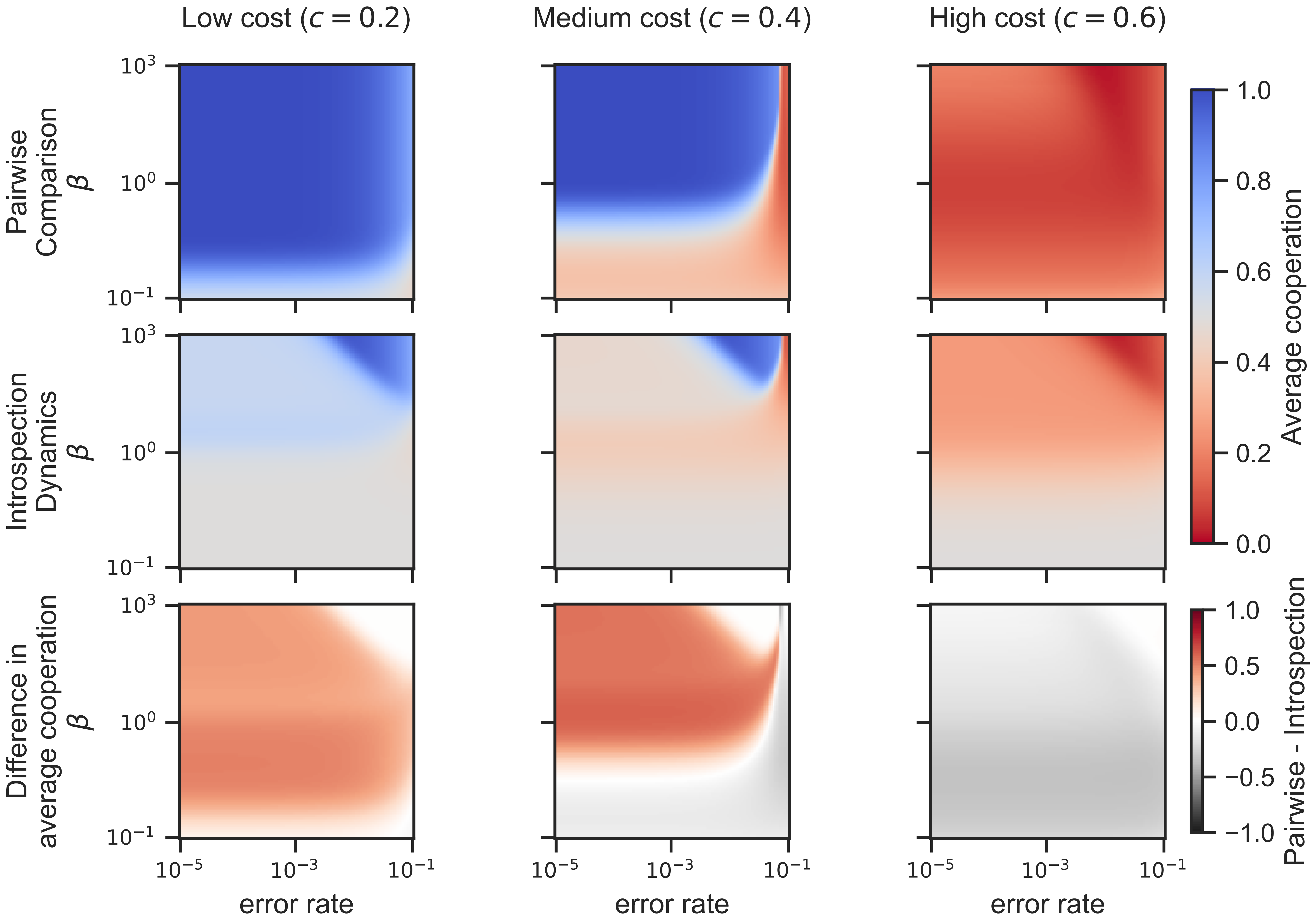


Social learning vs Individual learning

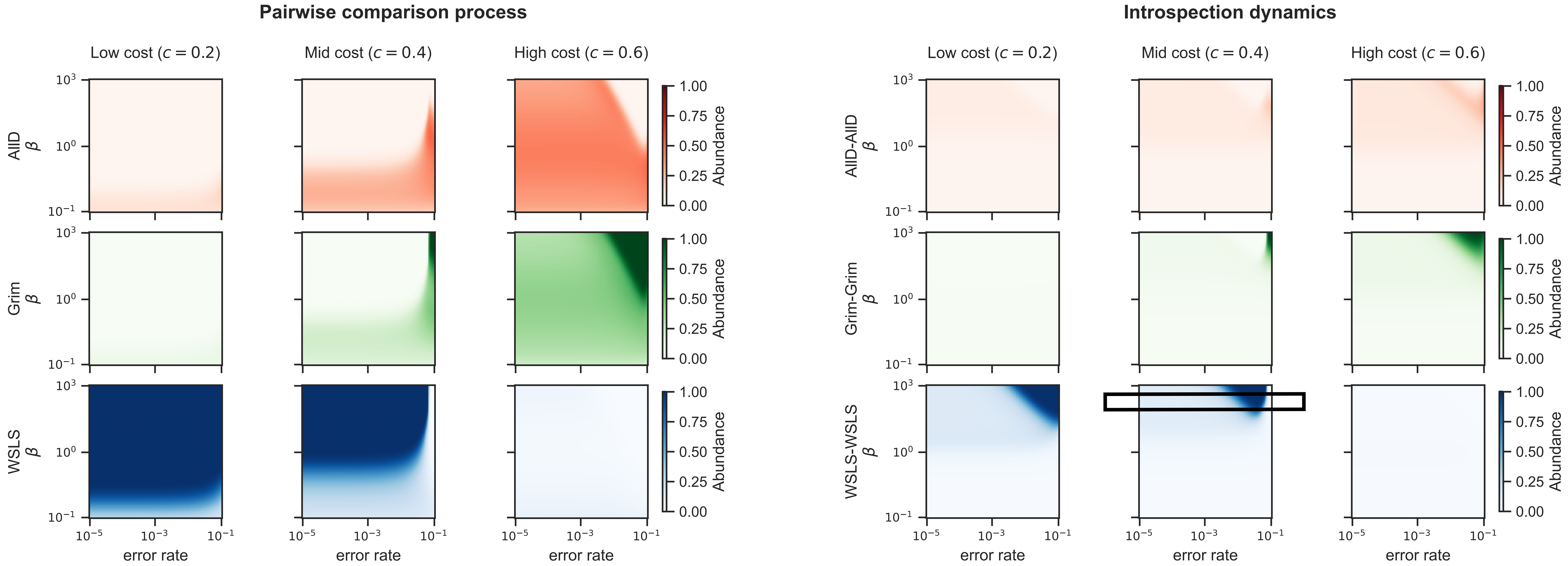


$1 - \mu$: **learns**

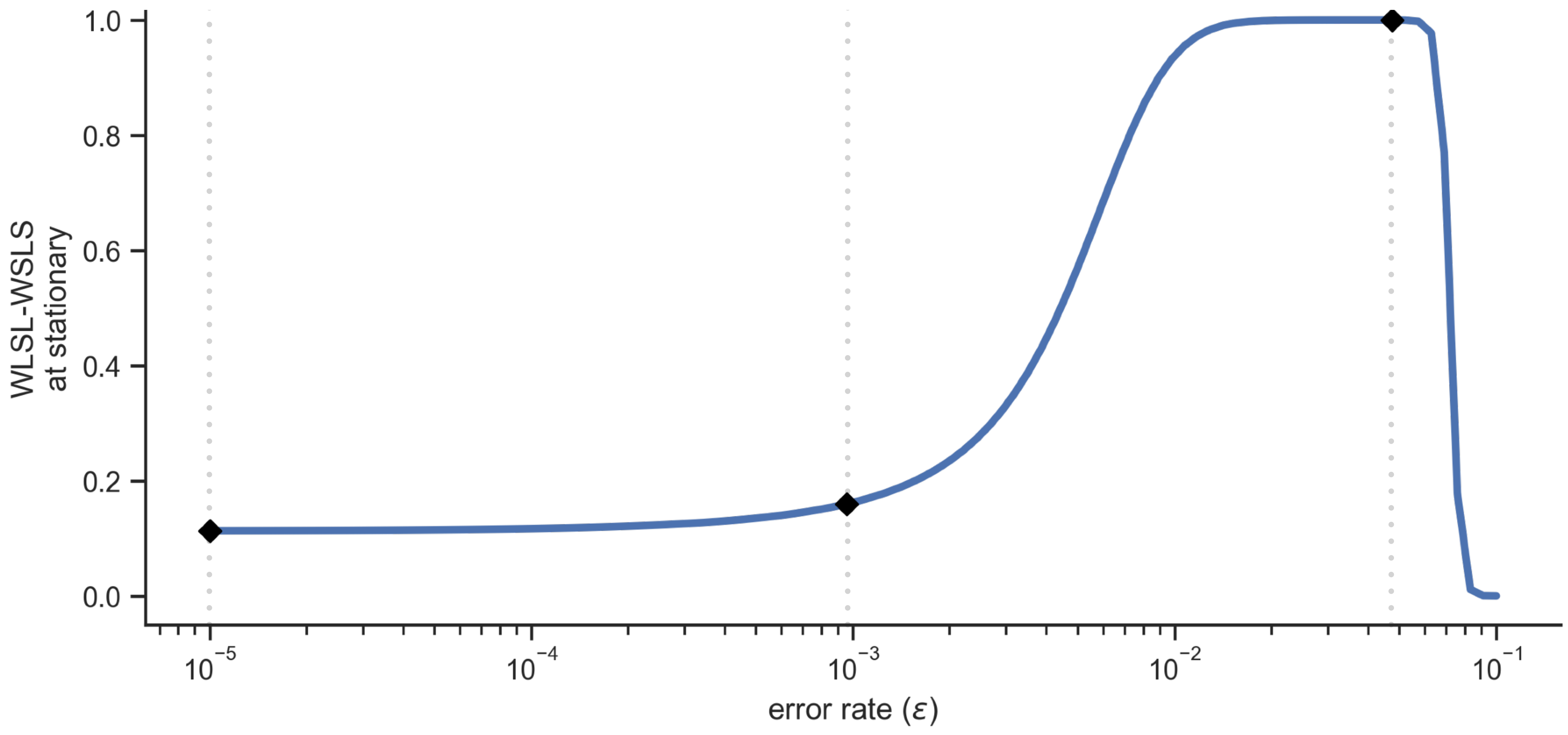
Social learning vs Individual learning



Social learning vs Individual learning

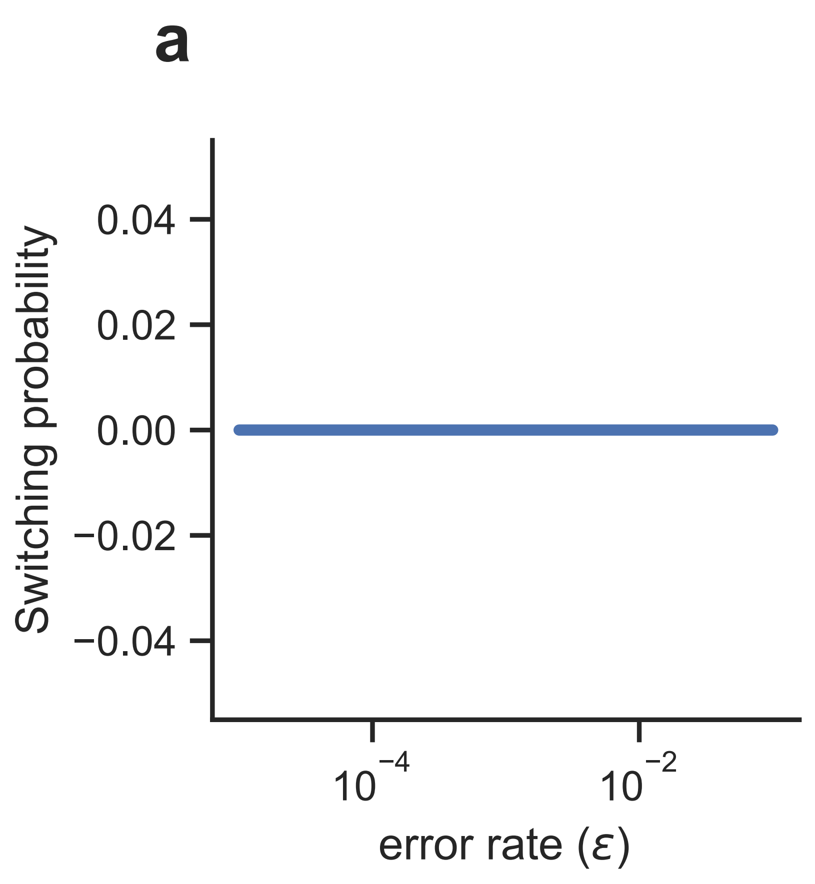


Social learning vs Individual learning

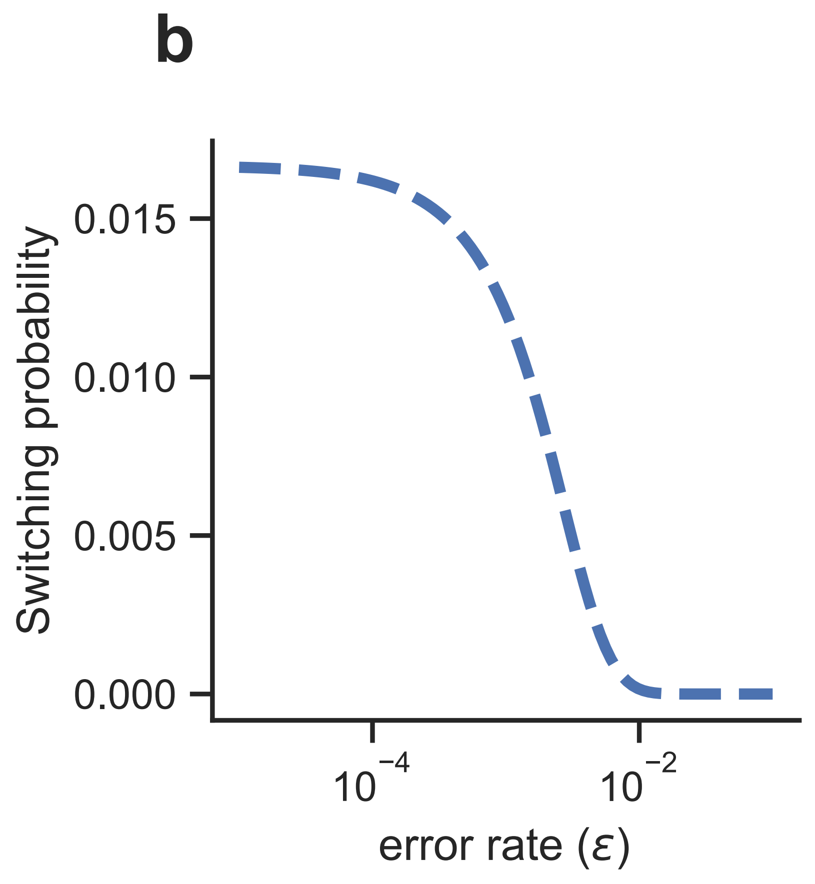


Social learning vs Individual learning

Pairwise comparison process

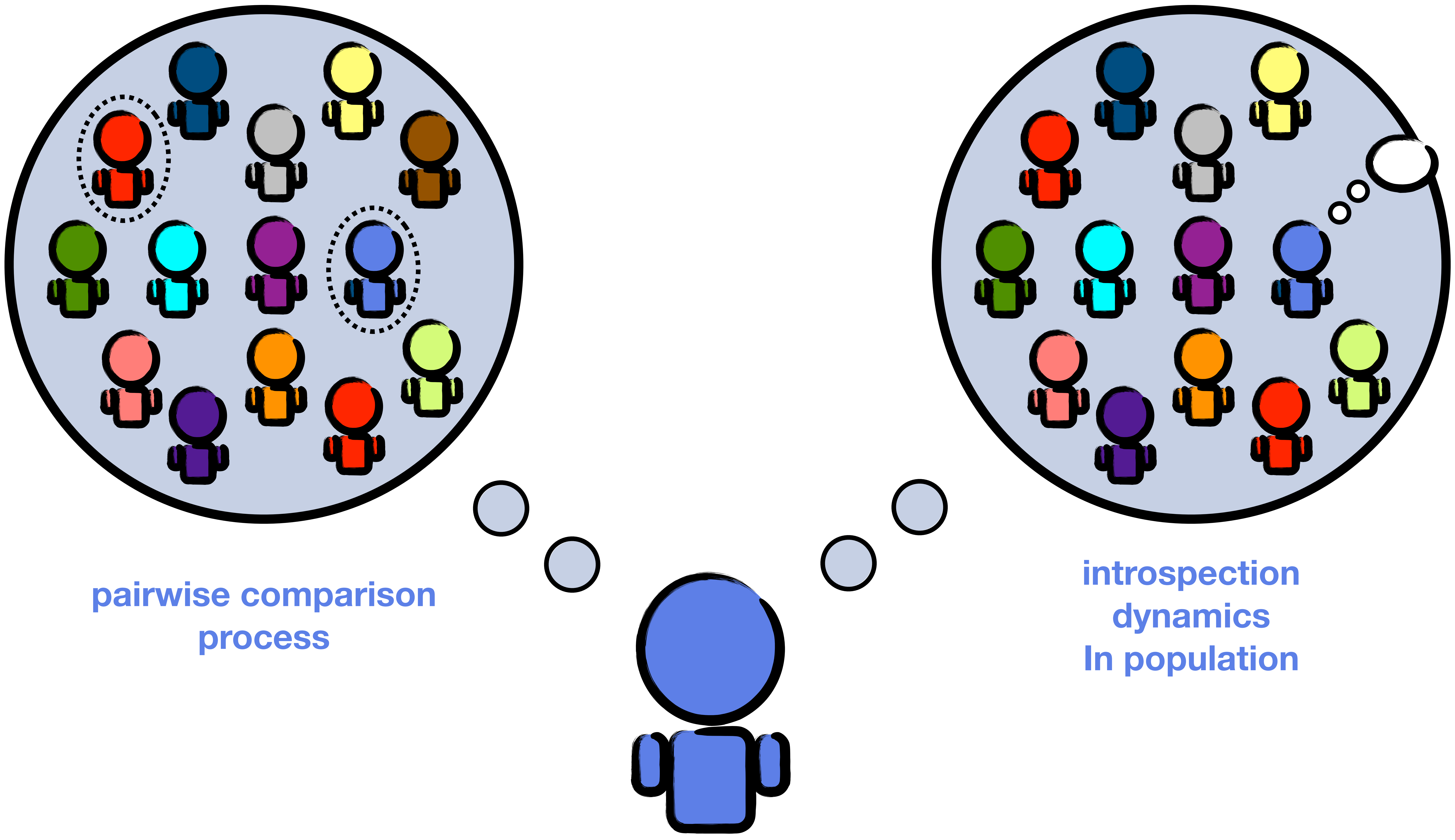


Introspection dynamics



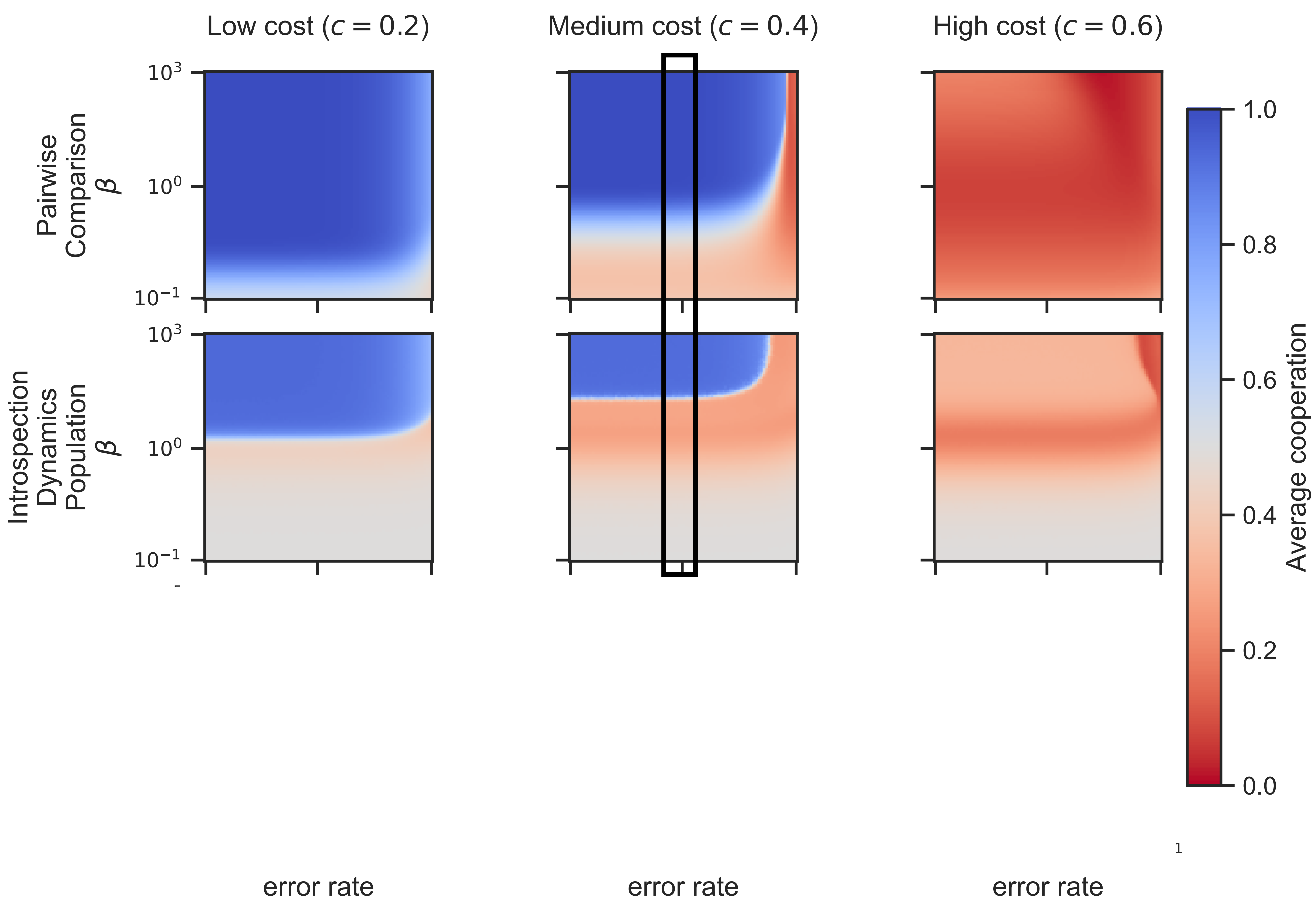
Self payoffs are important

Social learning vs Individual learning

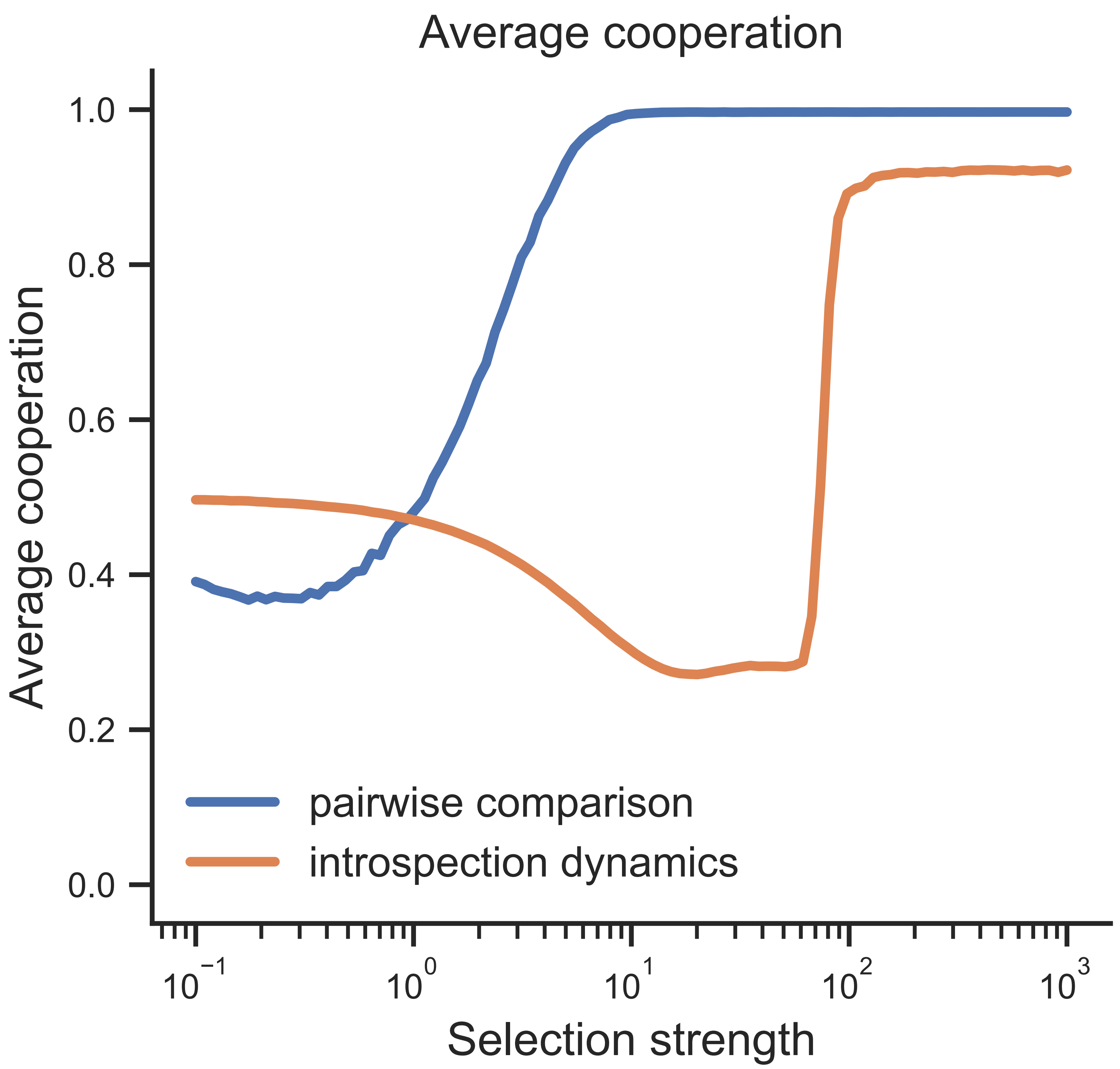


$1-\mu$: **learns**

Social vs Individual learning



Social vs Individual learning



SUMMARY

- Evolutionary outcomes depend strongly on modelling assumptions
- Memory, information, and learning shape direct reciprocity
- More memory expands cooperative equilibrium spaces and promotes cooperation
- Cooperation can still evolve under limited payoff information
- Different learning dynamics lead to qualitatively different evolutionary outcomes
- Restricting strategy spaces changes which strategies and equilibria emerge

Publications

[1] Conditional cooperation with longer.
https://doi.org/10.1073/pnas.2420125121

[2] Can I afford to remember less than you?
https://doi.org/10.1016/j.econlet.2025.112300

[3] Evolution of reciprocity with limited payoff memory.
https://doi.org/10.1098/rspb.2023.2493

[4] Characterisation of reactive Nash equilibria in repeated additive games.
https://arxiv.org/abs/2606.27653

THANK YOU!

Collaborators





Nikoleta-v3

http://nikoleta-v3.github.io