Limited information and the effects on the evolution of cooperation

EGAI 2025

Nikoleta E. Glynatsi

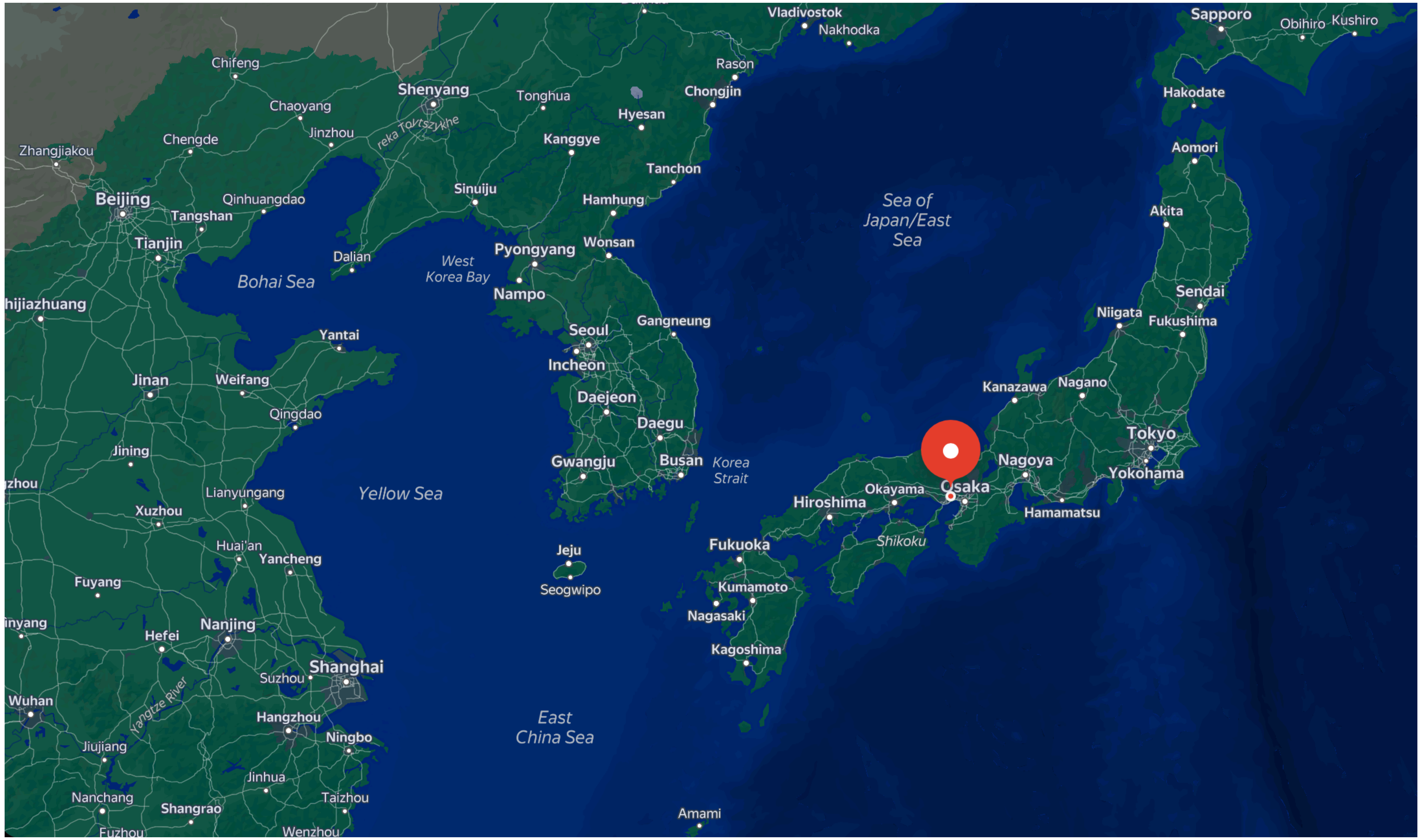
CARDIFF
UNIVERSITY

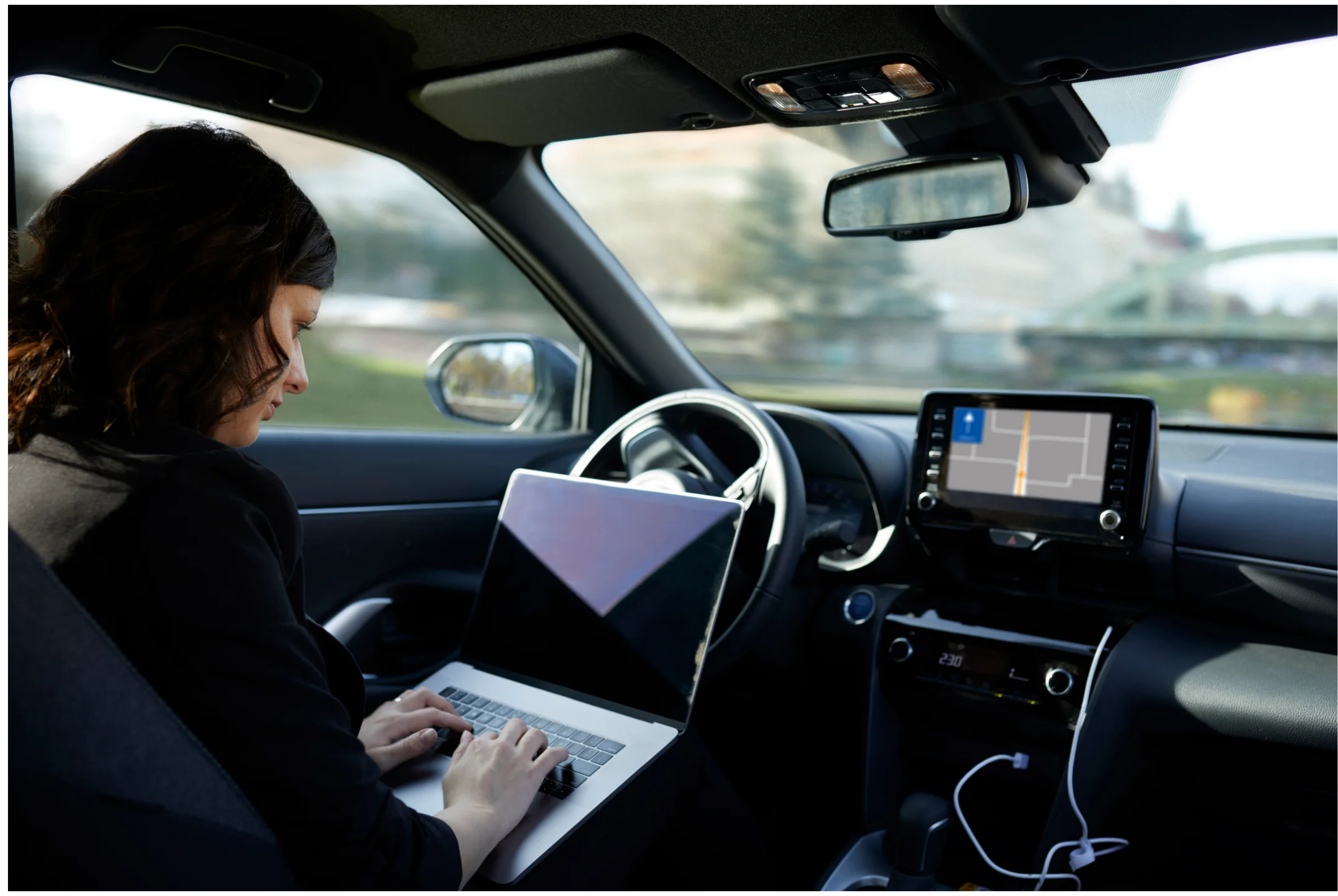
PRIFYSGOL
CAERDYDD

RIKEN

Discrete Event Simulation Team

Mathematical Social Science





COOPERATION

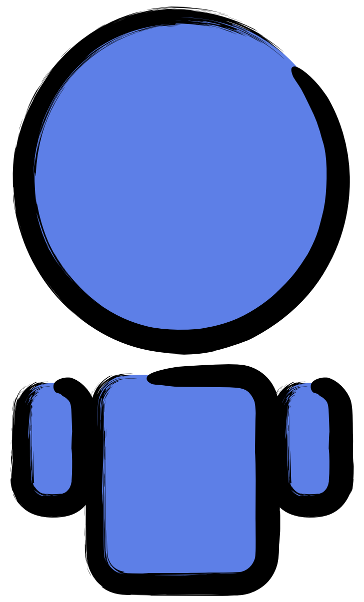
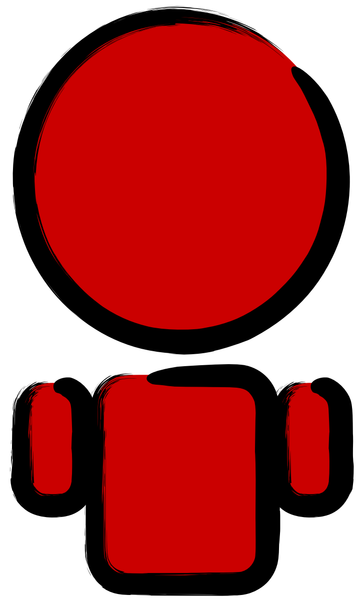
PRISONER'S DILEMMA

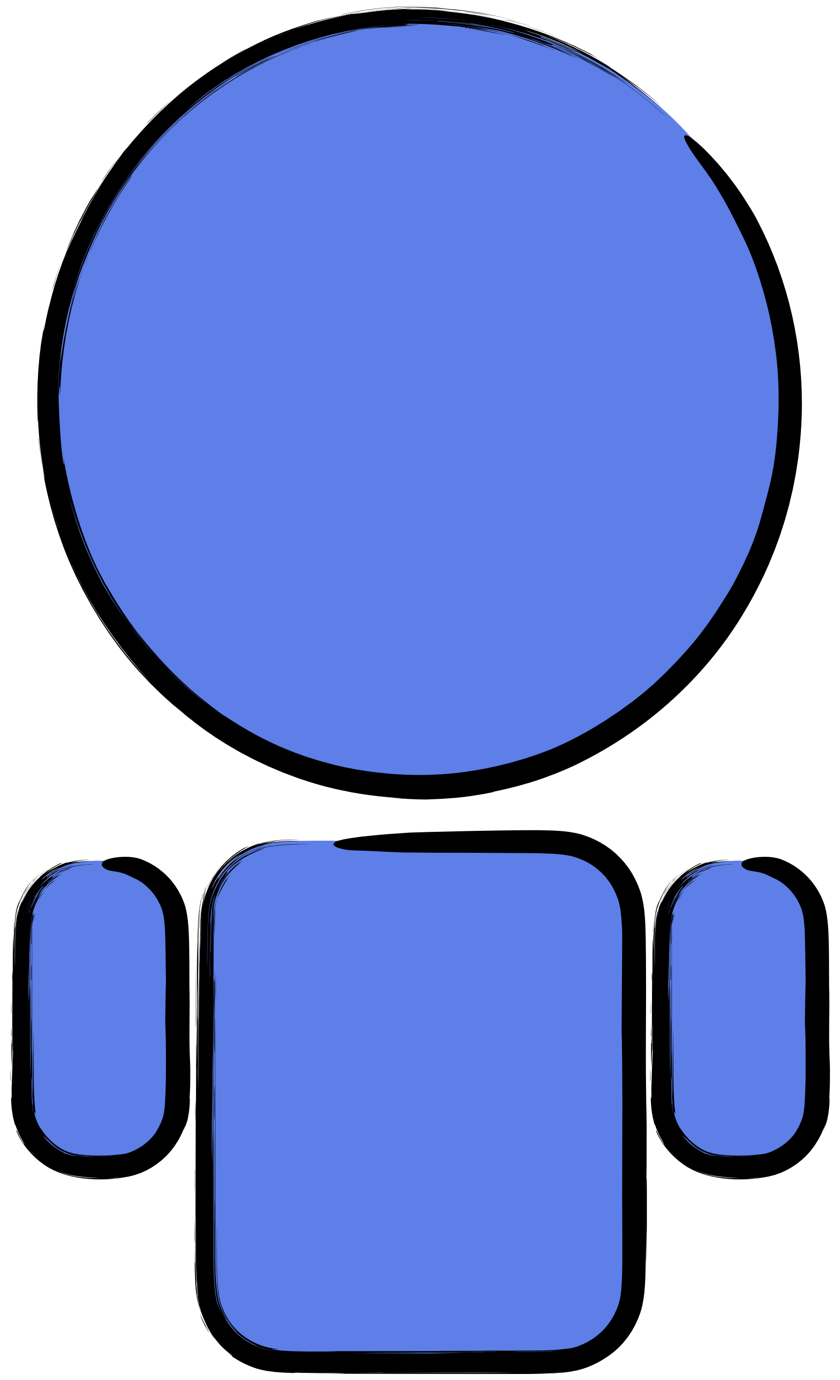
$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} b-c & -c \\ b & \boxed{0} \end{pmatrix} \end{array}$$

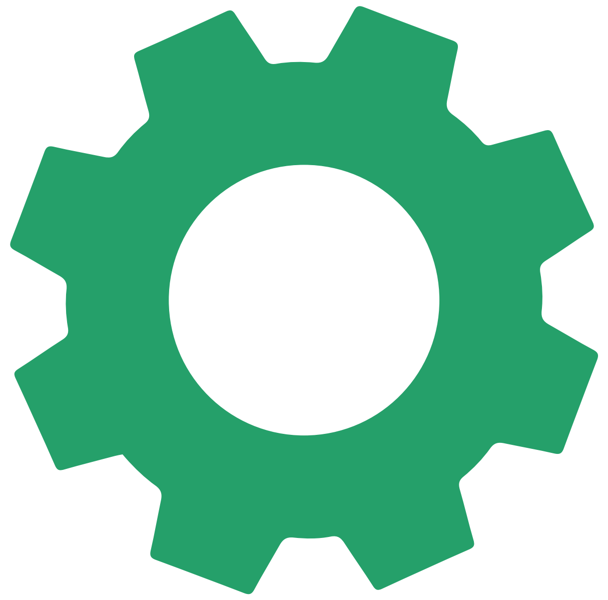
Nash Equilibrium

$$b > c > 0$$

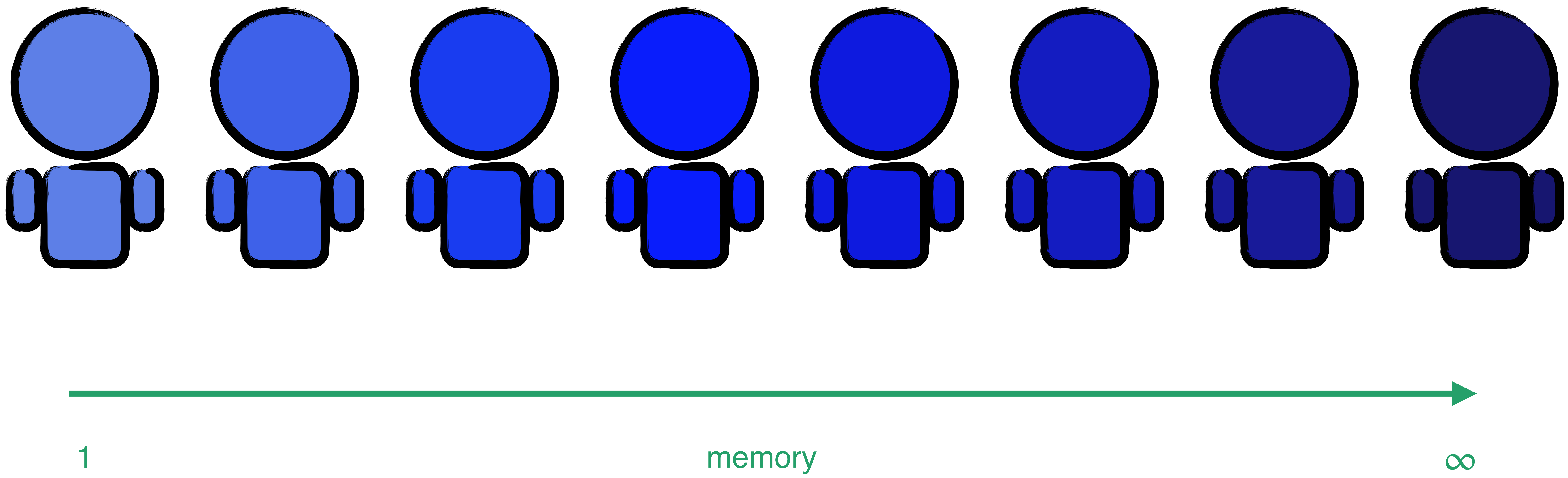
DIRECT RECIPROCITY

	1	2	3		n
	$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$	$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$	$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$	$\cdots$	$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$
	D	C	C	$\cdots$	C
	D	D	C	$\cdots$	C

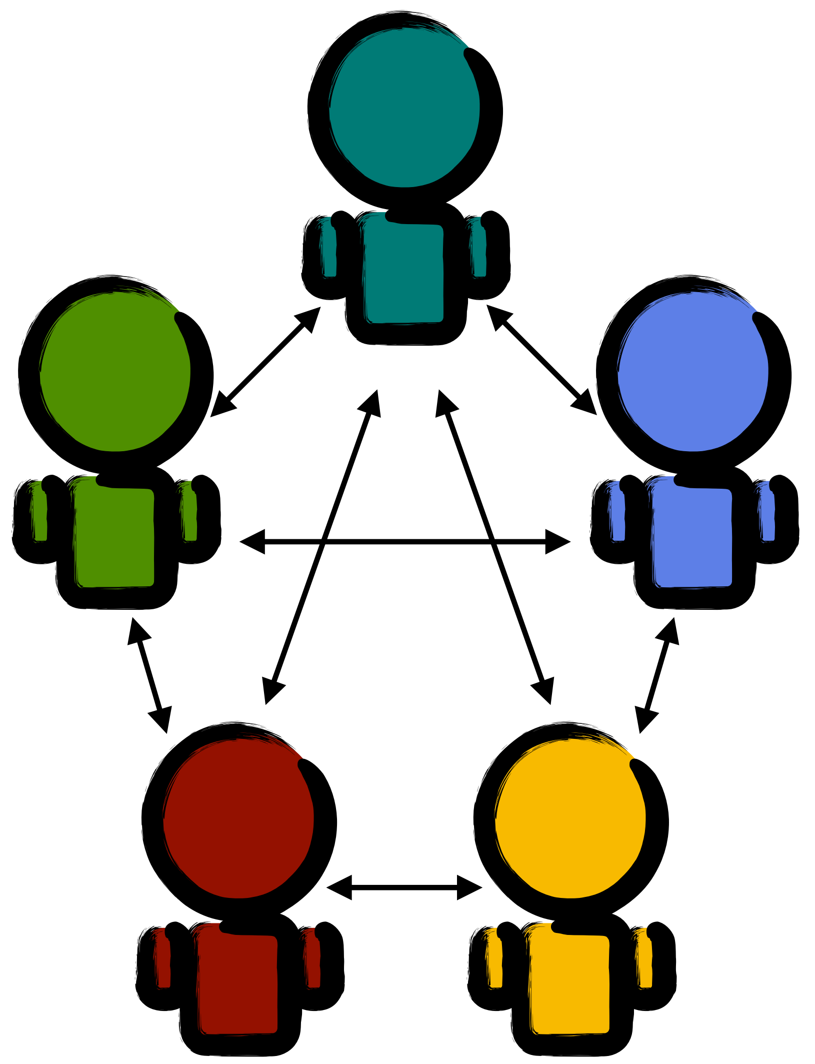




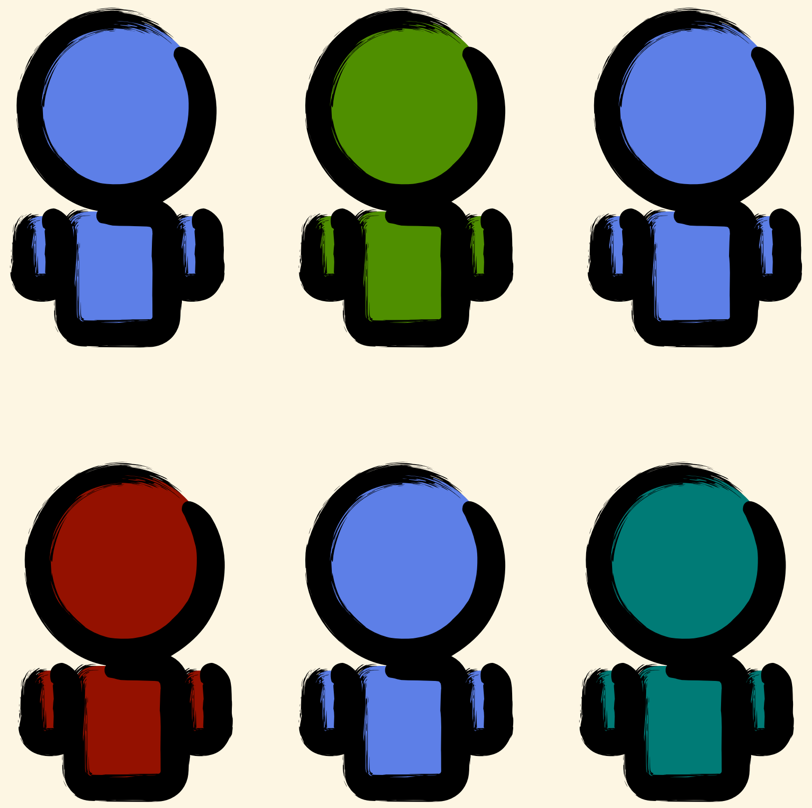
remember &
process information


1
memory
∞

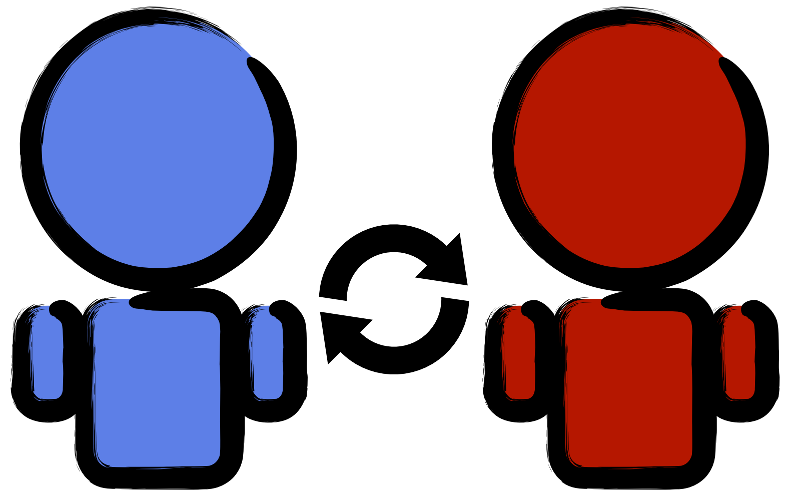
Strategies in computer tournaments



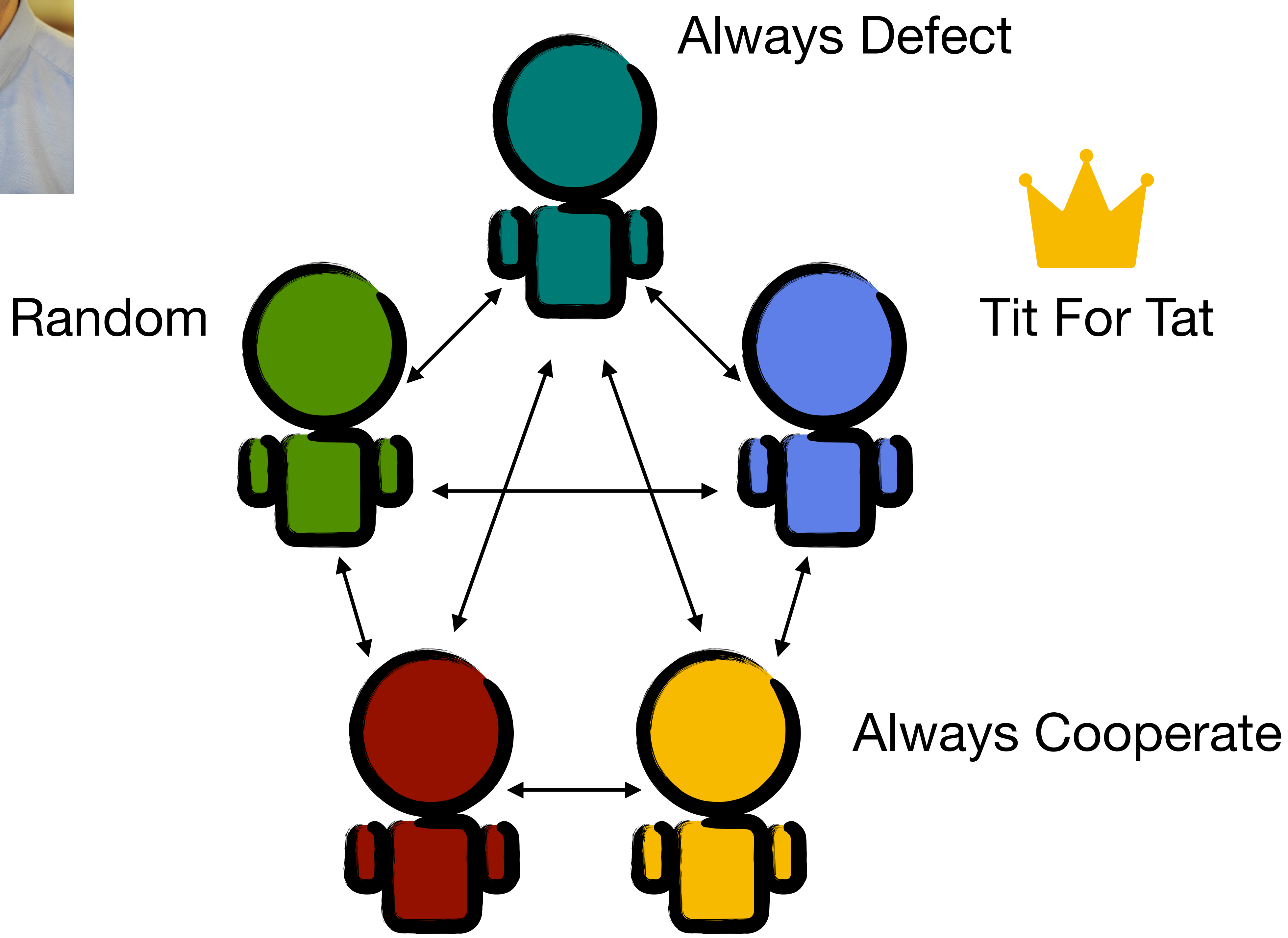
Learning in populations



Strategies in repeated interactions



STRATEGIES IN COMPUTER TOURNAMENTS



Most strategies are quite simple rules

↓

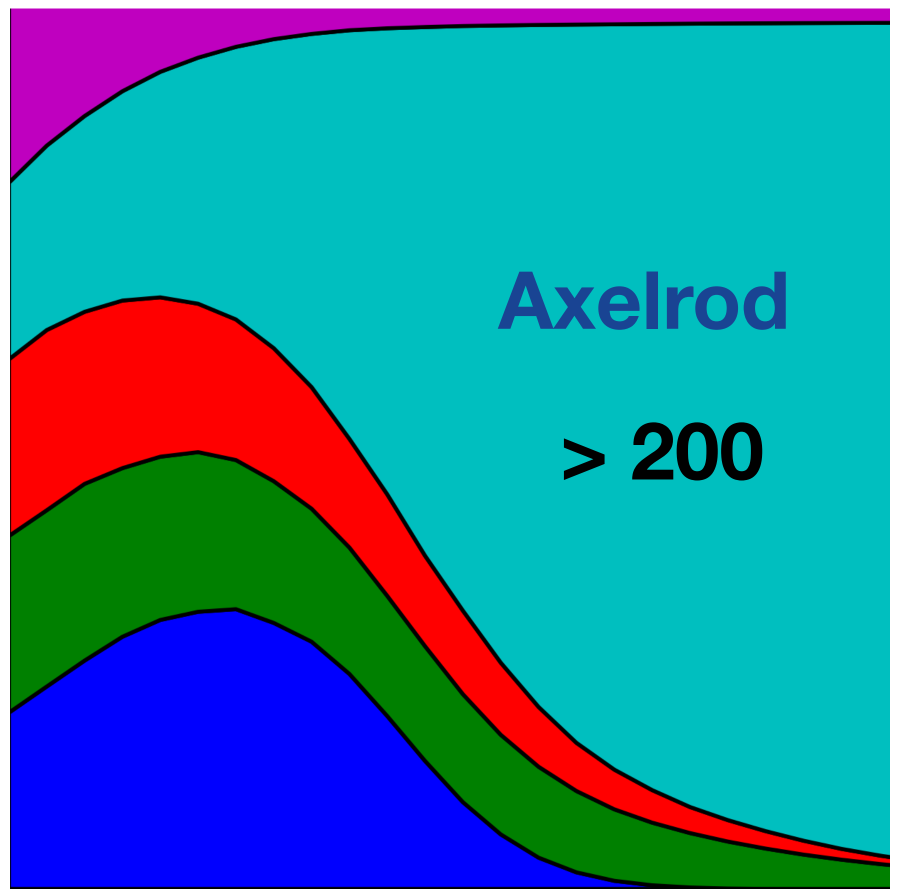
Interpretability

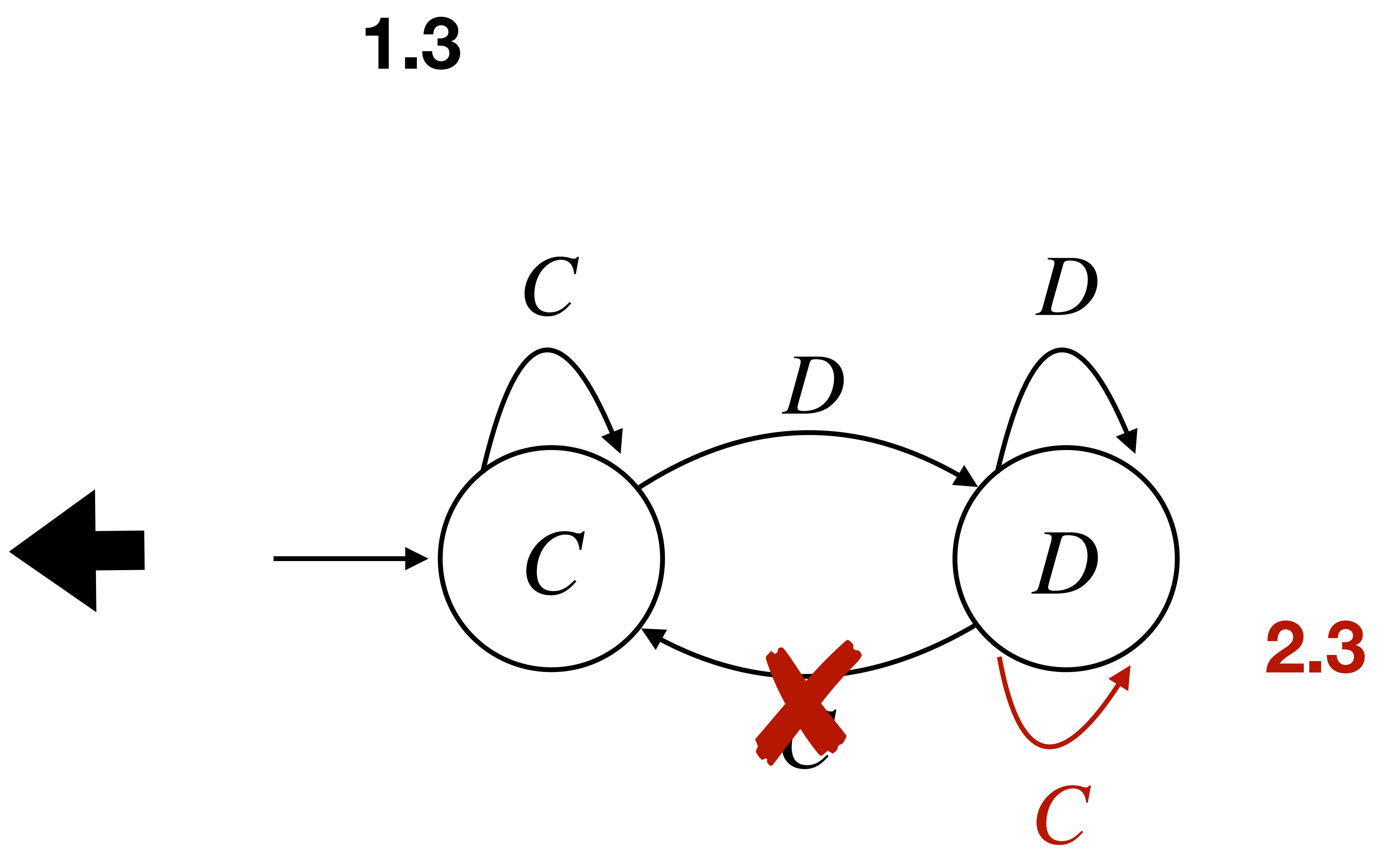
Introduce complex strategies

↓

Understand properties of them

STRATEGIES IN COMPUTER TOURNAMENTS





* tutorial tomorrow

	mean	std	min	5%	25%	50%	75%	95%	max
EvolvedLookerUp2_2_2*	2.955	0.010	2.915	2.937	2.948	2.956	2.963	2.971	2.989
Evolved HMM 5*	2.954	0.014	2.903	2.931	2.945	2.954	2.964	2.977	3.007
Evolved FSM 16*	2.952	0.013	2.900	2.930	2.943	2.953	2.962	2.973	2.993
PSO Gambler 2_2_2*	2.938	0.013	2.884	2.914	2.930	2.940	2.948	2.957	2.972
Evolved FSM 16 Noise 05*	2.919	0.013	2.874	2.898	2.910	2.919	2.928	2.939	2.965
PSO Gambler 1_1_1*	2.912	0.023	2.805	2.874	2.896	2.912	2.928	2.950	3.012
Evolved ANN 5*	2.912	0.010	2.871	2.894	2.905	2.912	2.919	2.928	2.945
Evolved FSM 4*	2.910	0.012	2.867	2.889	2.901	2.910	2.918	2.929	2.943
Evolved ANN*	2.907	0.010	2.865	2.890	2.900	2.908	2.914	2.923	2.942
PSO Gambler Mem1*	2.901	0.025	2.783	2.858	2.884	2.901	2.919	2.942	2.994
Evolved ANN 5 Noise 05*	2.864	0.008	2.830	2.850	2.858	2.865	2.870	2.877	2.891
DBS	2.857	0.009	2.823	2.842	2.851	2.857	2.863	2.872	2.899
Winner12	2.849	0.008	2.820	2.836	2.844	2.850	2.855	2.862	2.874
Fool Me Once	2.844	0.008	2.818	2.830	2.838	2.844	2.850	2.857	2.882
Omega TFT: 3, 8	2.841	0.011	2.800	2.822	2.833	2.841	2.849	2.859	2.882

https://doi.org/10.1371/journal.pone.0188046.t002

Top performing strategies in a tournament with over 200 strategies.

STRATEGIES IN COMPUTER TOURNAMENTS

- From the training emerged strategies that were cooperative but also took advantage of simple strategies

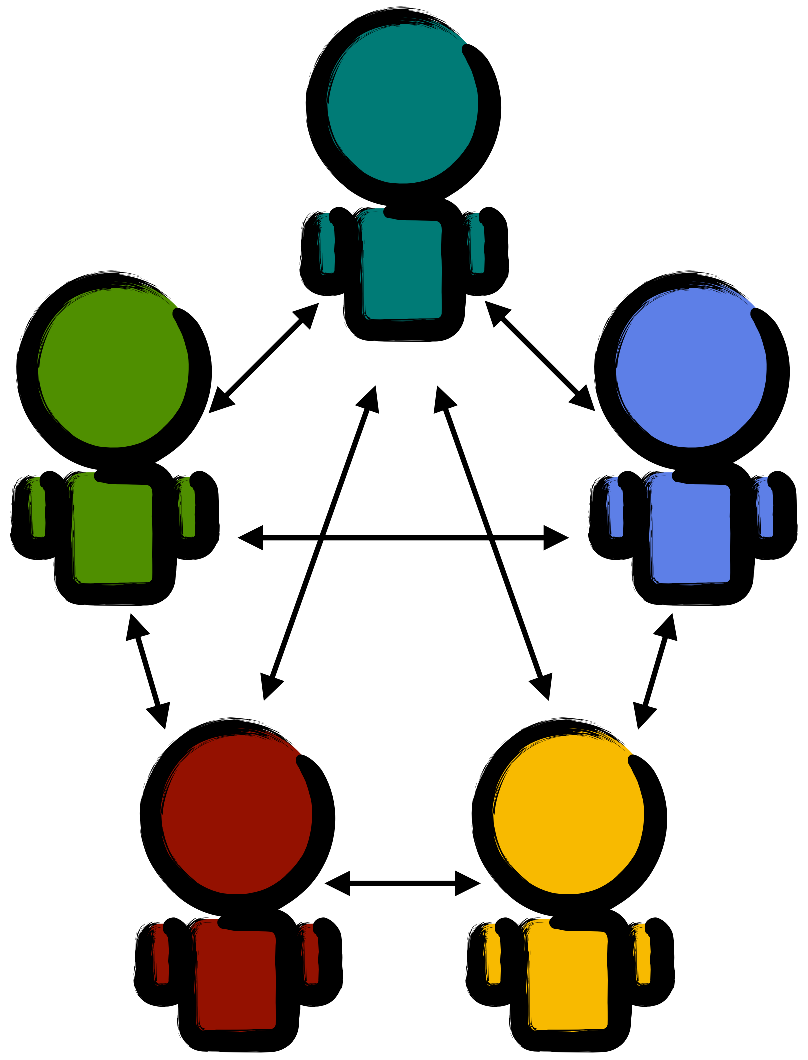
- Strategies trained in environments with errors were more adaptable

[1] Reinforcement learning produces dominant strategies for the iterated prisoner's dilemma.
https://doi.org/10.1371/journal.pone.0188046

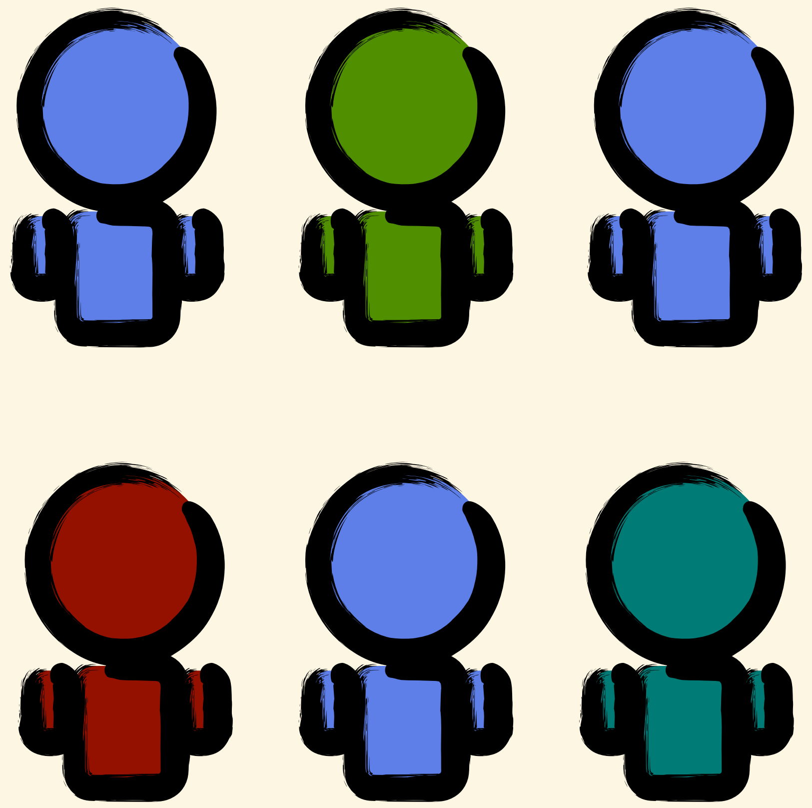
[2] Evolution reinforces cooperation with the emergence of self-recognition mechanisms.
https://doi.org/10.1371/journal.pone.0204981

[3] Properties of winning iterated prisoner's dilemma strategies.
https://doi.org/10.1371/journal.pcbi.1012644

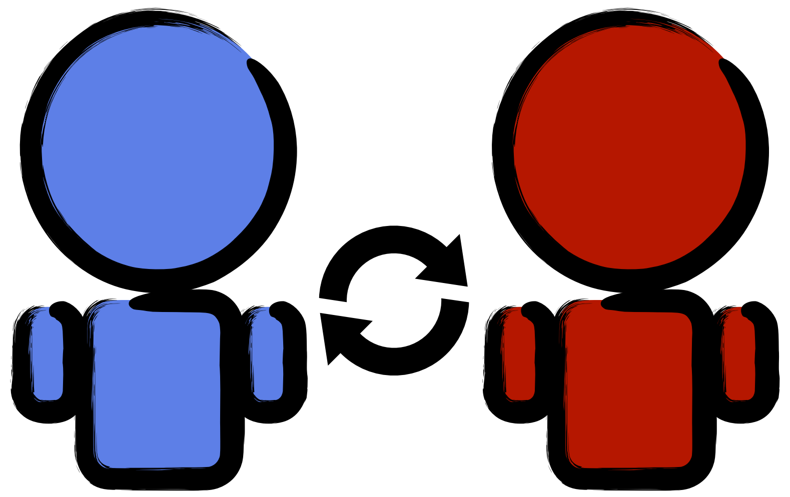
Strategies in computer tournaments



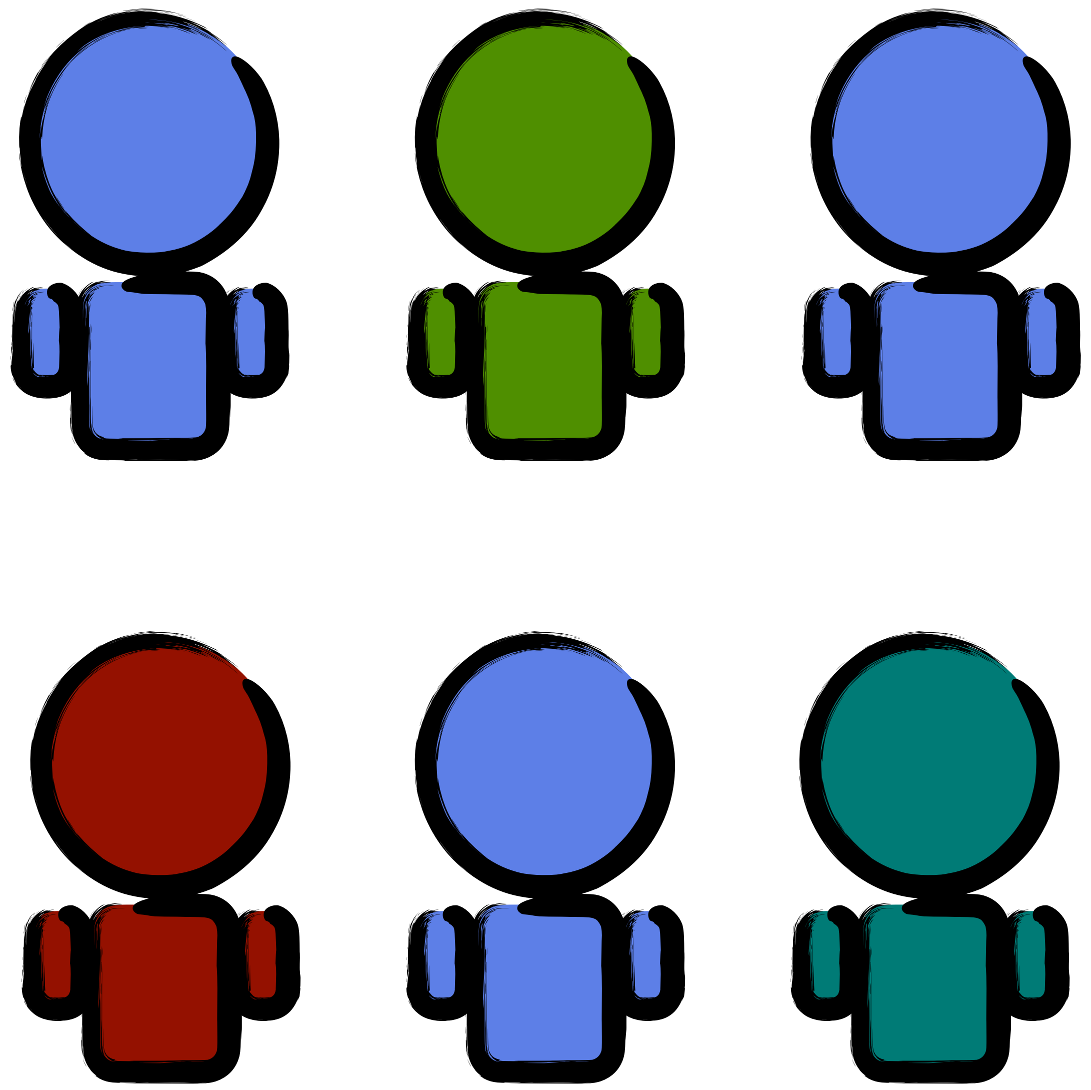
Learning in populations

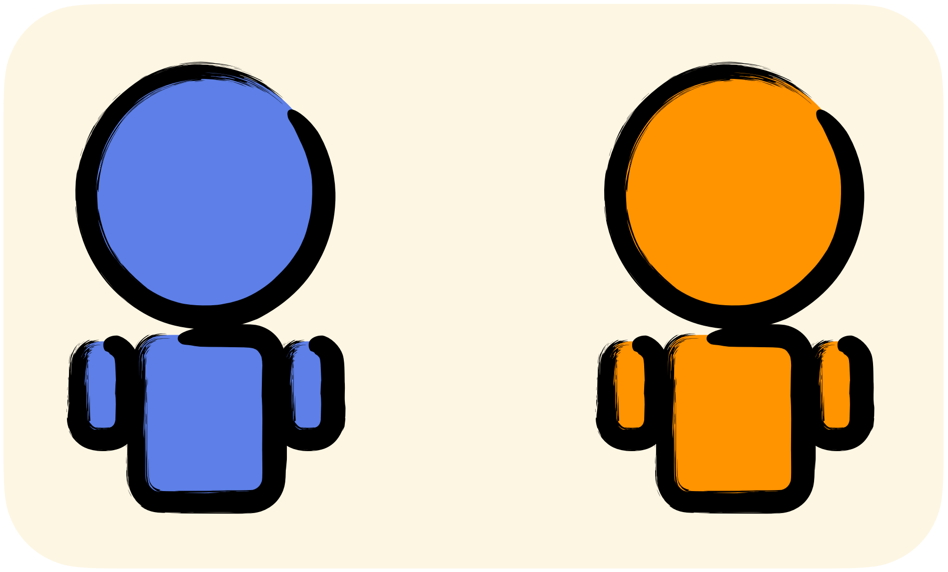


Strategies in repeated interactions

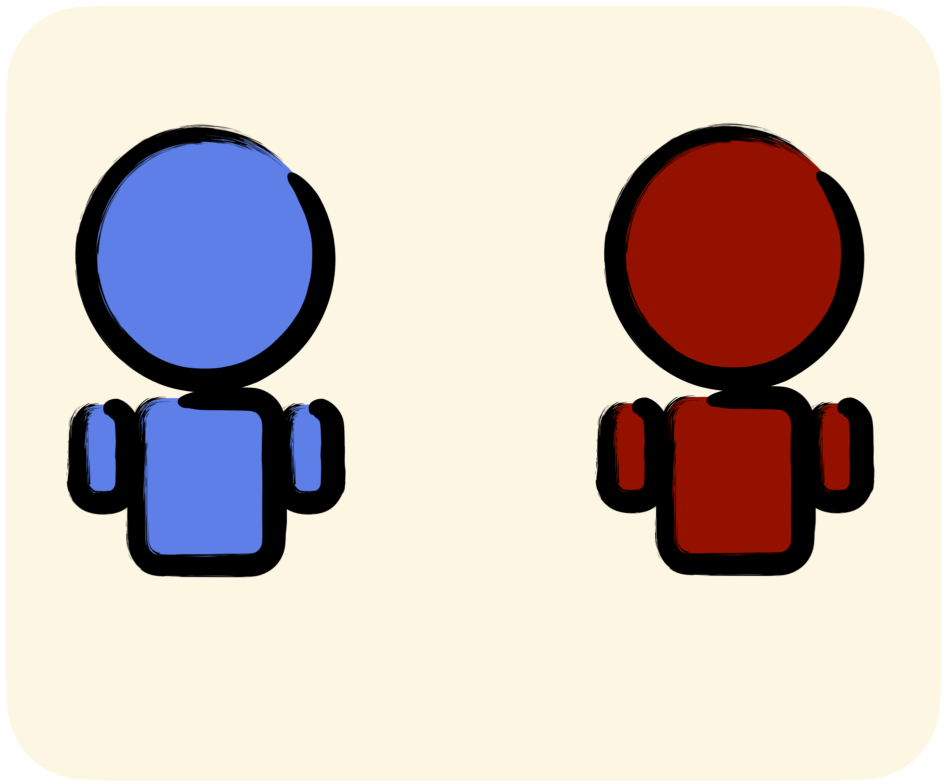


LEARNING IN POPULATIONS





μ: **mutation**

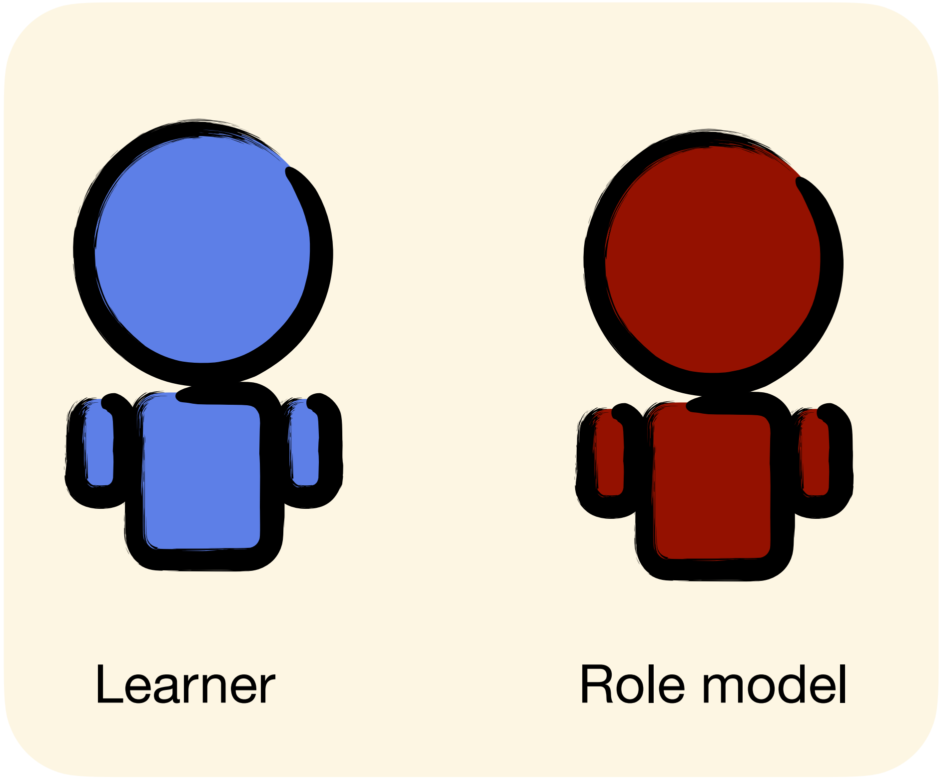


$$\phi(\pi_L, \pi_{RM}) = \frac{1}{1 + e^{-\beta(\pi_{blue} - \pi_{red})}}$$

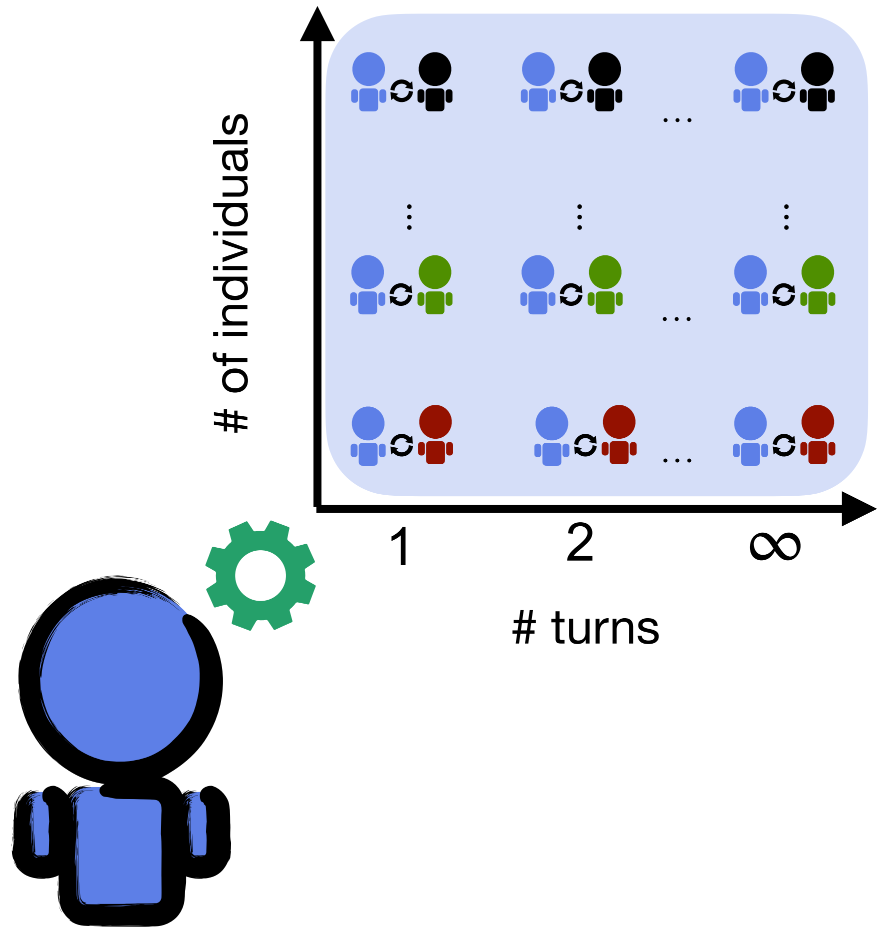
β: **strength of selection**

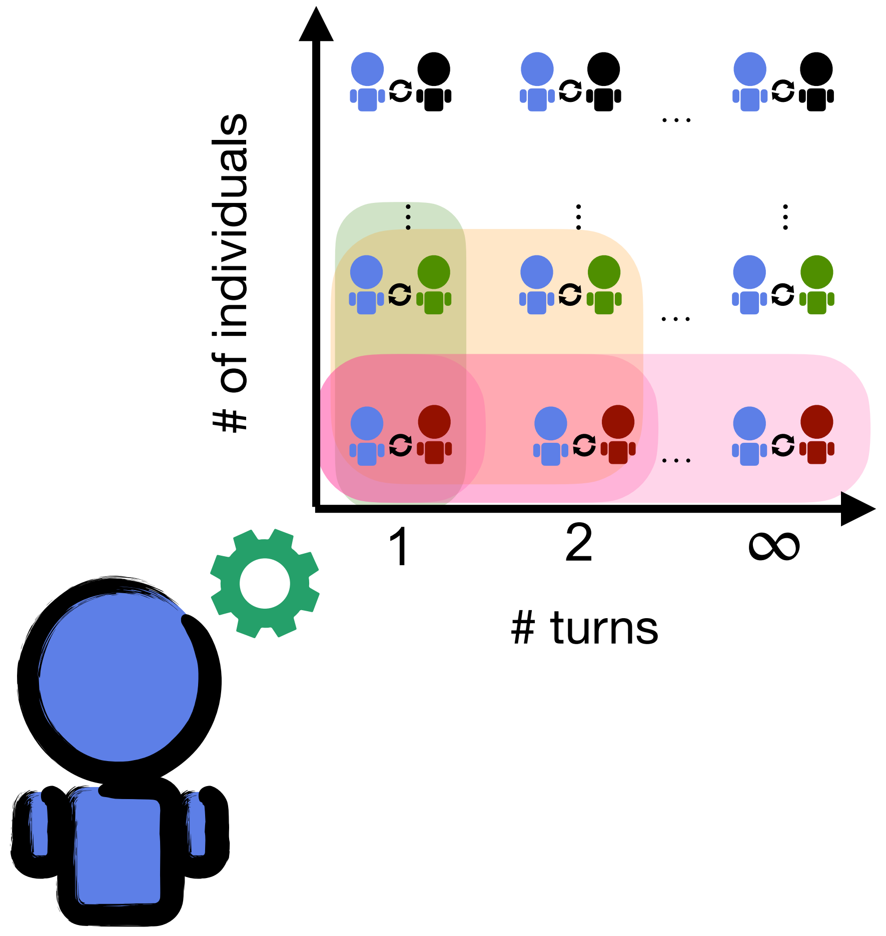
$1 - \mu$: **imitaton**

LEARNING IN POPULATIONS

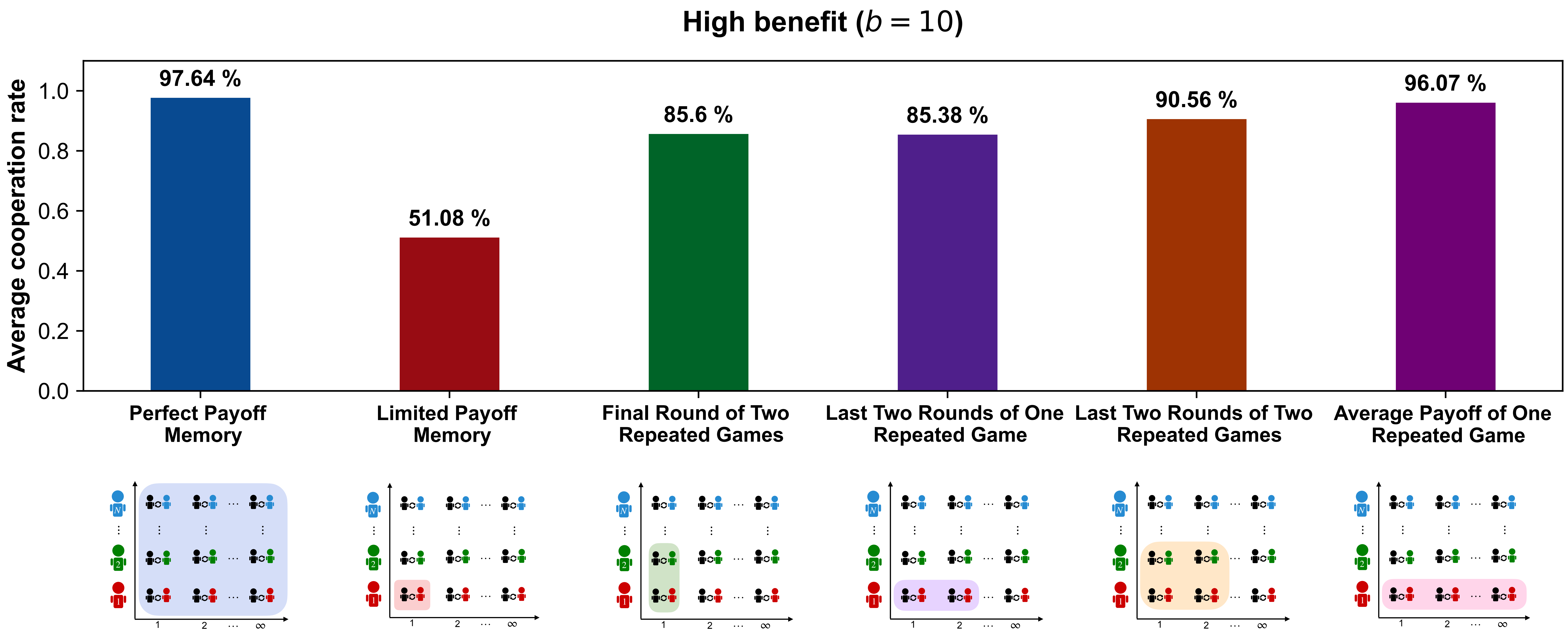


$1-\mu$: **imitaton**





LEARNING IN POPULATIONS

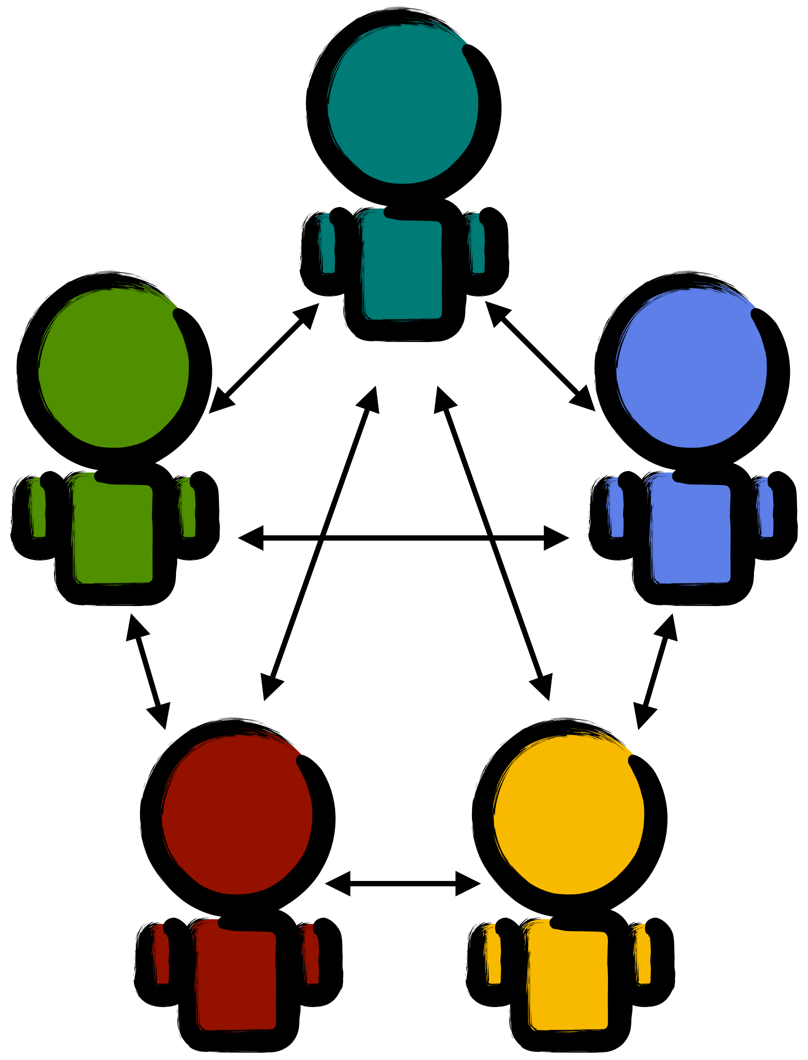


LEARNING IN POPULATIONS

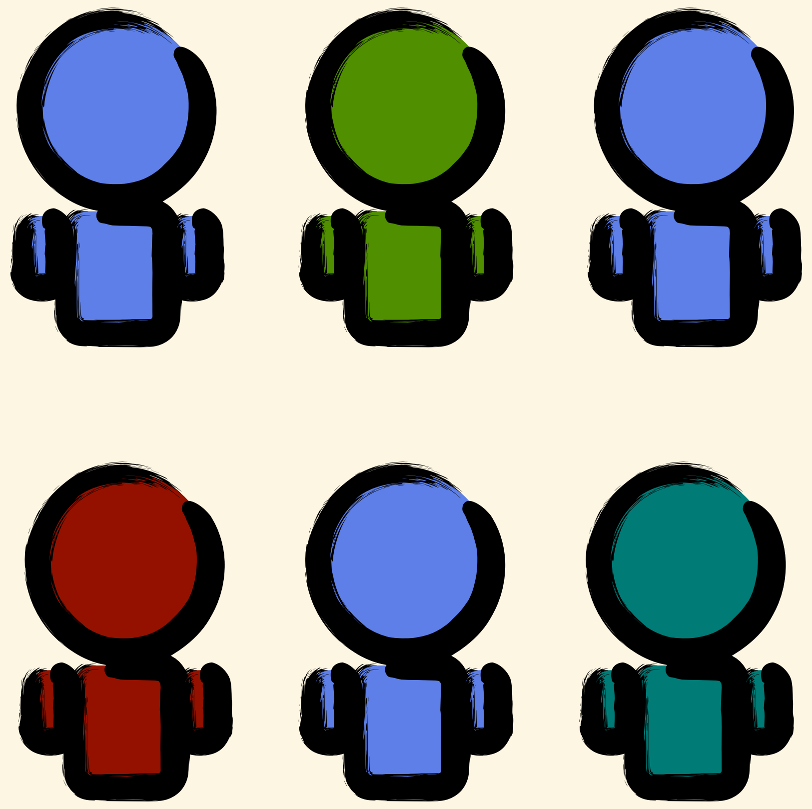
- Cooperation still evolves even with limited memory
- As individuals remember two or three recent interactions, the cooperation rates approach the classical limit

[4] Evolution of reciprocity with limited payoff memory.
https://doi.org/10.1098/rspb.2023.2493

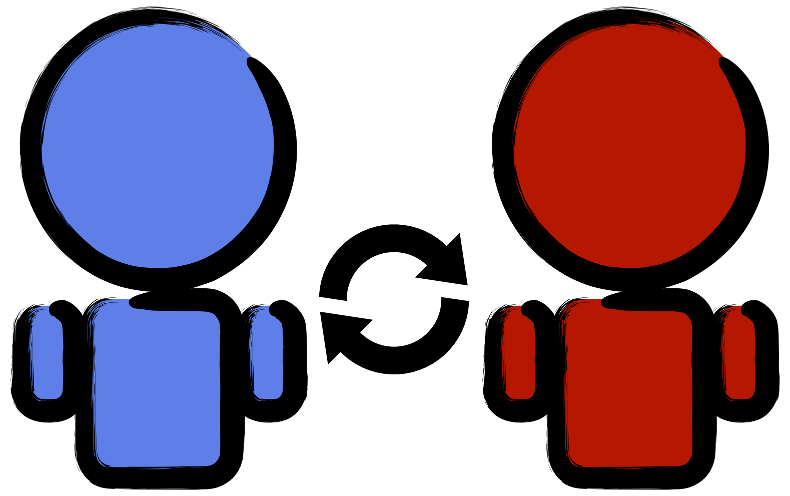
Strategies in computer tournaments



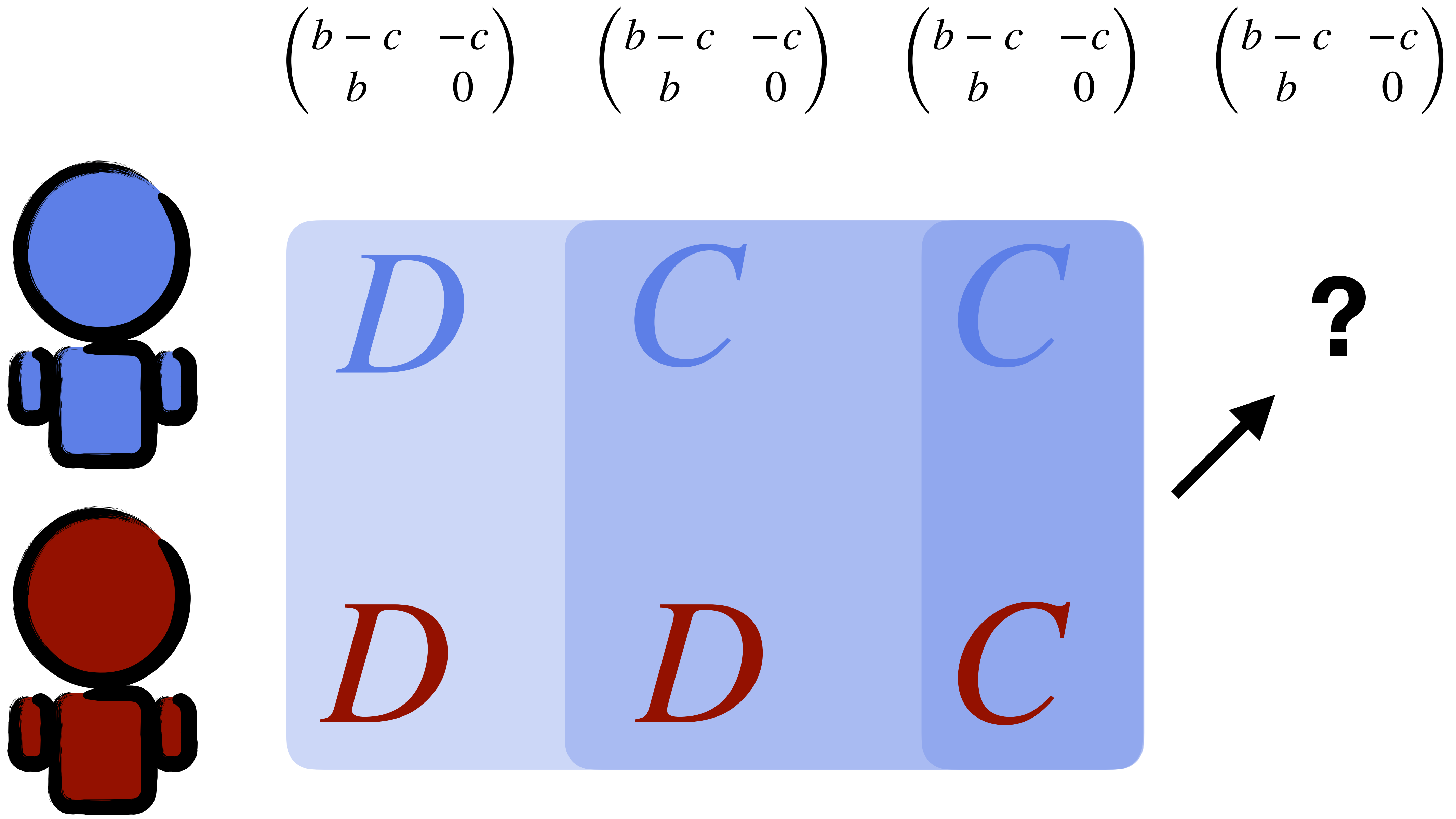
Learning in populations



Strategies in repeated interactions



STRATEGIES IN REPEATED GAMES

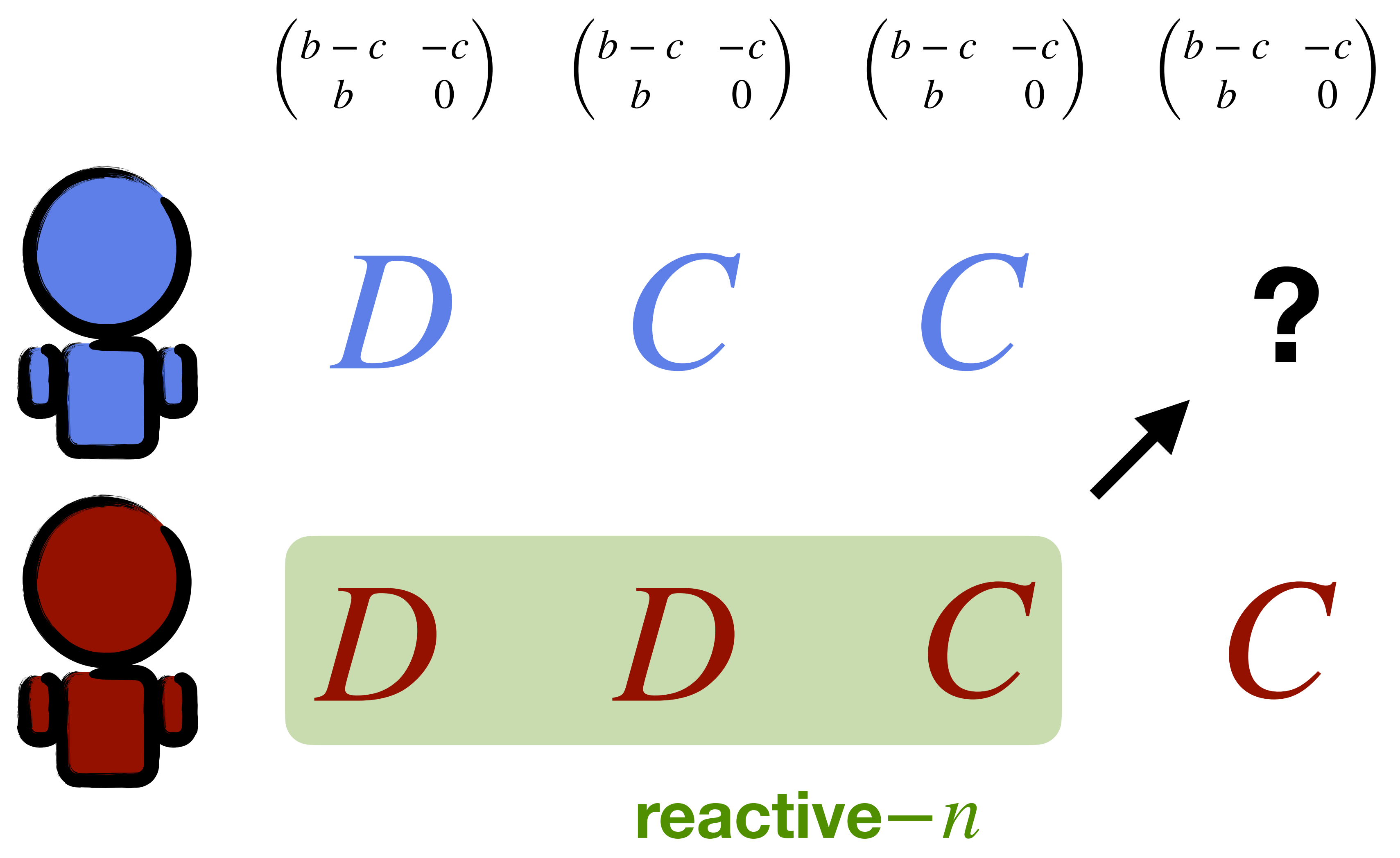


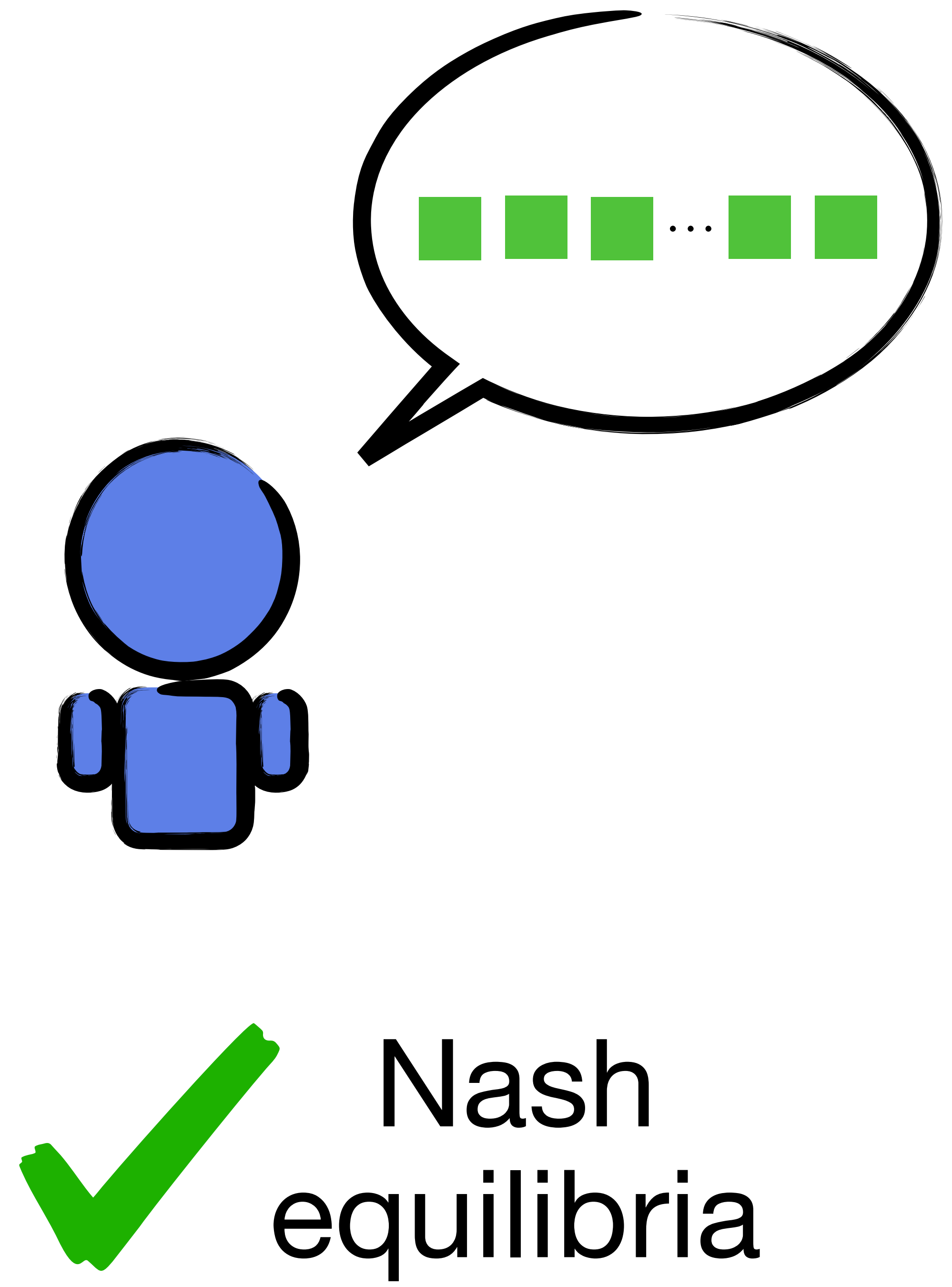
memory-2

Nash equilibria among memory-1 strategies

STRATEGIES IN REPEATED GAMES

Can we say anything about Nash equilibria in repeated games with higher memory than $n = 1$?





STRATEGIES IN REPEATED GAMES

Definition 1.

A reactive-n strategy can be defined as 2^n-dimensional vector $\mathbf{p} = (p_{\mathbf{h}^{-\mathbf{i}}})_{\mathbf{h}^{-\mathbf{i}} \in H^{-i}}$ with $0 \leq p_{\mathbf{h}^{-\mathbf{i}}} \leq 1$ where $\mathbf{h}^{-\mathbf{i}}$ refers to an n-history of the co-player from the space of all possible co-player histories.

Examples.

Tit for tat $(1,0)$ Random $(\frac{1}{2}, \frac{1}{2})$

A reactive-1 strategy can be defined as: $\mathbf{p} = (p_C, p_D)$

A reactive-2 strategy can be defined as: $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$

A reactive-3 strategy can be defined as: $\mathbf{p} = (p_{CCC}, p_{CCD}, p_{CDC}, p_{CDD}, p_{DCC}, p_{DCD}, p_{DDC}, p_{DDD})$

Definition 2.

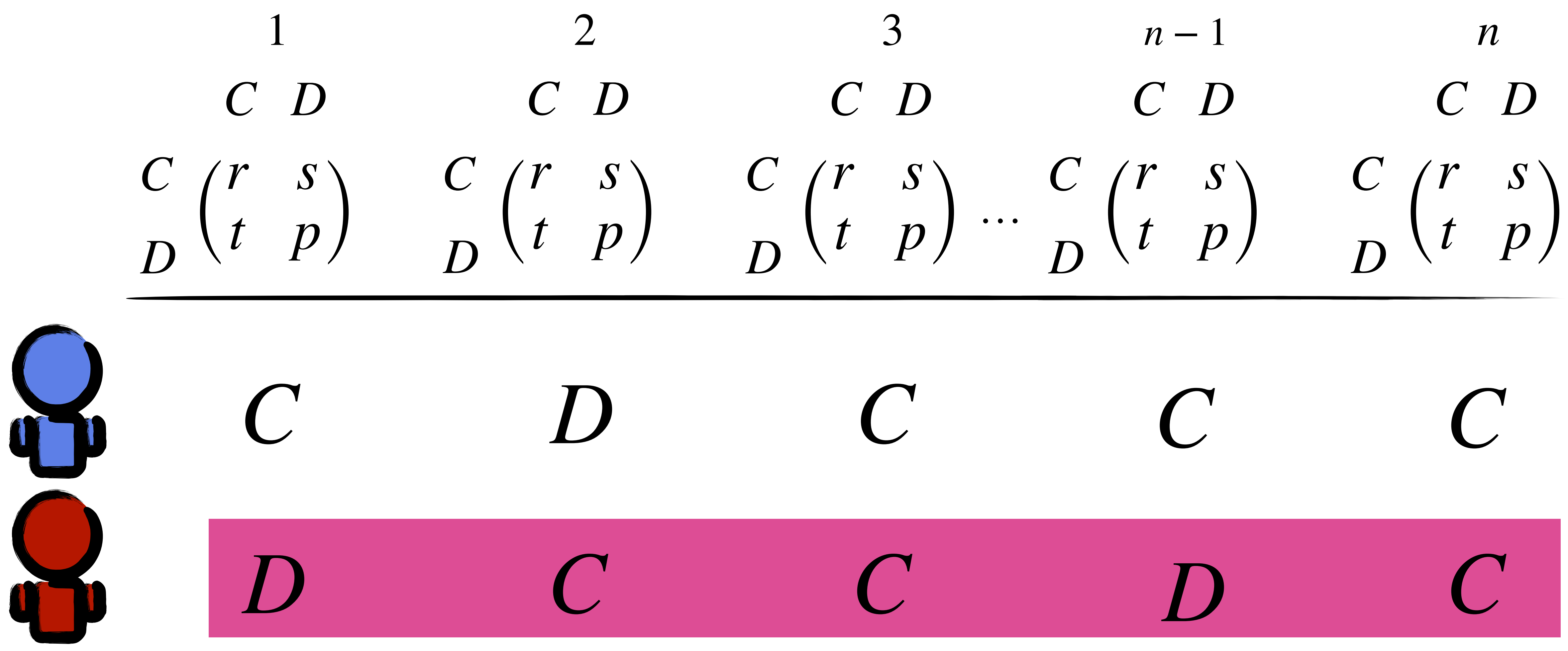
A strategy $\mathbf{p}$ for a repeated game is a Nash equilibrium if it is a best response to itself.

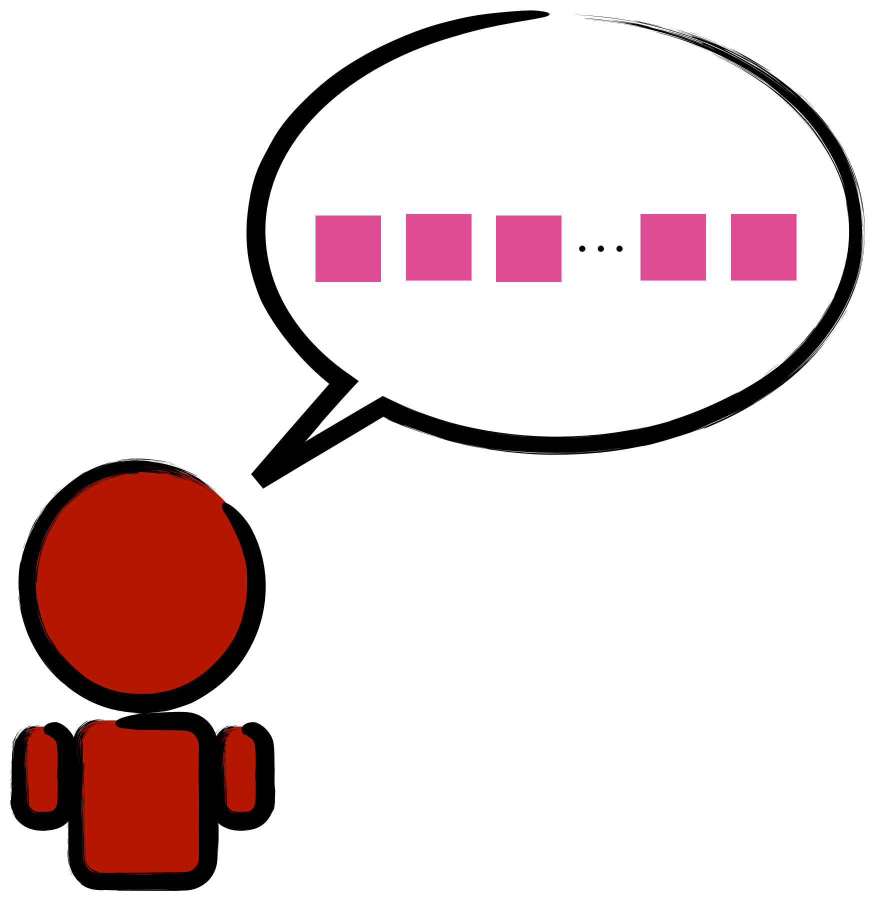
That is $\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\sigma, \mathbf{p})$ for all other strategies σ.


p

STRATEGIES IN REPEATED GAMES

Theorem. A reactive strategy $\mathbf{p} \in \mathscr{R}_n$ is a Nash equilibrium if and only if $\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\tilde{\mathbf{p}}, \mathbf{p})$ for all pure self-reactive strategies $\tilde{\mathbf{p}}$.





self-reactive-n

STRATEGIES IN REPEATED GAMES

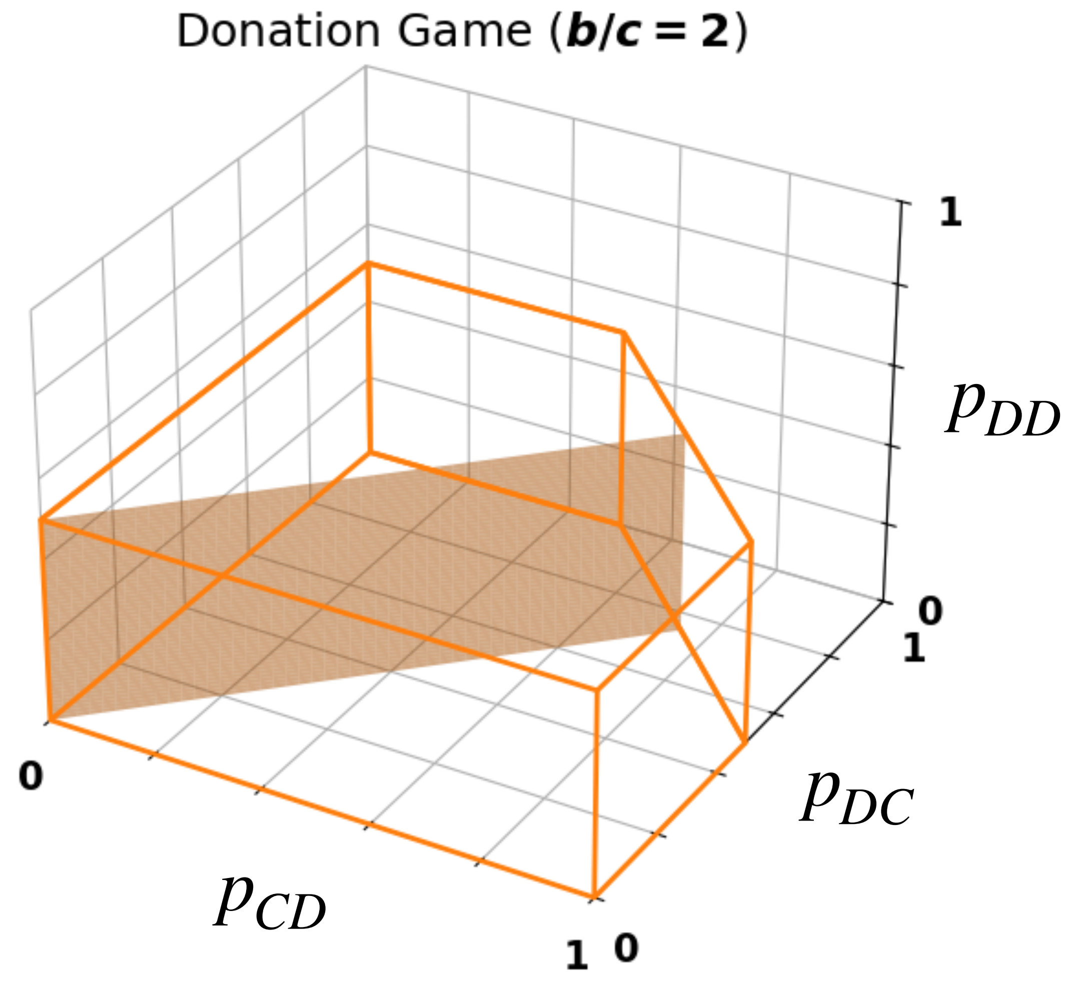
Theorem. A reactive strategy $\mathbf{p} \in \mathcal{R}_n$ is a Nash equilibrium if and only if $\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\tilde{\mathbf{p}}, \mathbf{p})$ for all pure self-reactive strategies $\tilde{\mathbf{p}}$.

$$\mathbf{p} = (p_{CCC}, p_{CCD}, p_{CDC}, p_{CDD}, p_{DCC}, p_{DCD}, p_{DDC}, p_{DDD}) \quad 256$$

STRATEGIES IN REPEATED GAMES

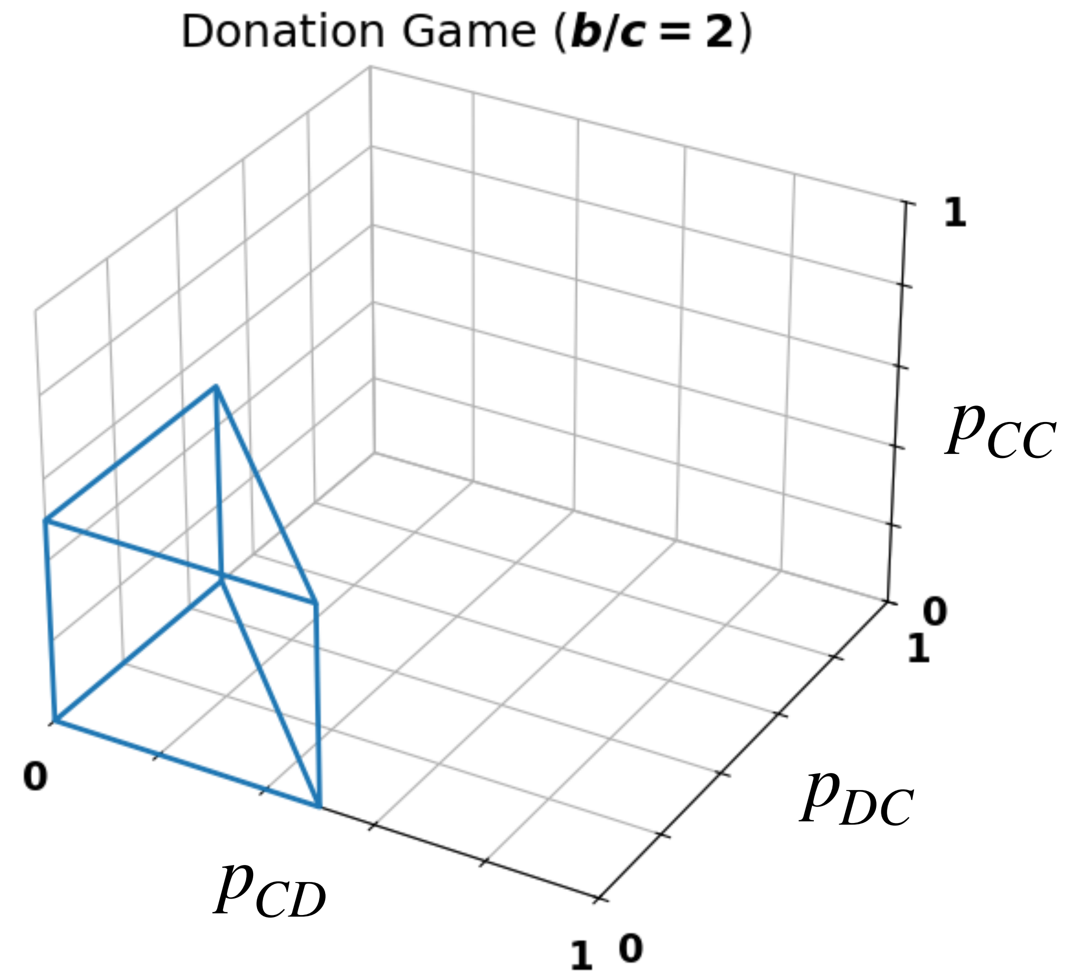
Theorem. A reactive-2 strategy $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ is a cooperative Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CC} = 1, \quad \frac{p_{CD} + p_{DC}}{2} < 1 - \frac{1}{2} \cdot \frac{c}{b}, \quad p_{DD} \leq 1 - \frac{c}{b}.$$



Theorem. A reactive-2 strategy $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ is a defective Nash equilibrium if and only if its entries satisfy the conditions,

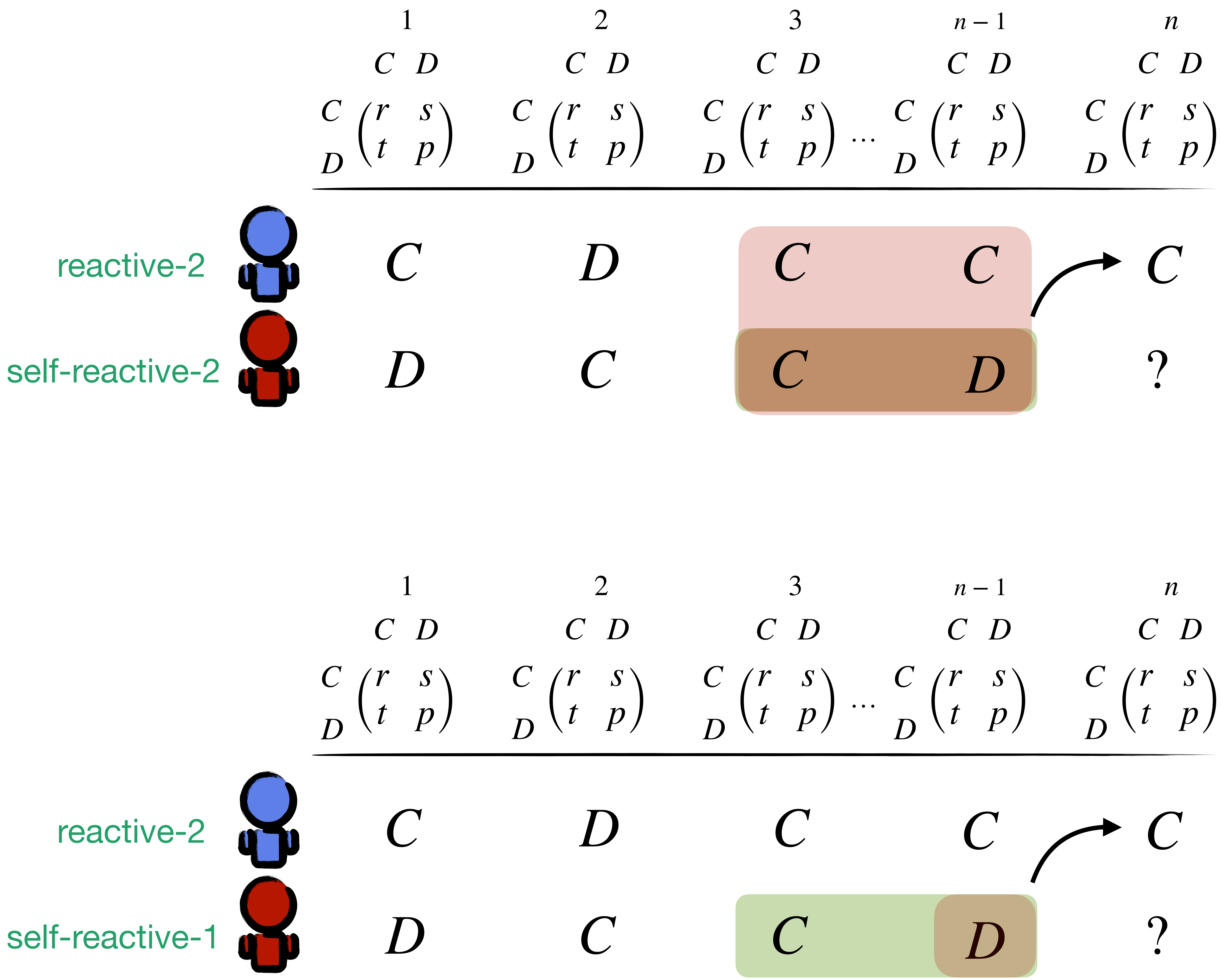
$$p_{CC} \leq \frac{c}{b} \quad \frac{p_{CD}+p_{DC}}{2} \leq \frac{c}{2b}, \quad p_{DD} = 0.$$



STRATEGIES IN REPEATED GAMES

- Algorithm to verify whether a given reactive-n strategy is an equilibrium.
- It's not just that having more memory gains you nothing. You also gain nothing from having more information.
- Fully characterize cooperative & defective equilibria for $n = 2$ and $n = 3$.

[5] Conditional cooperation with longer
https://doi.org/10.1073/pnas.2420125121



reactive-2
self-reactive-2
reactive-2
self-reactive-1

Theorem. Let $\mathbf{p} \in \mathscr{R}_n$ be a reactive-n strategy and the game additive. Then there exist a pure self-reactive-$(n-1)$ strategy $\tilde{\mathbf{p}}$ that is a best response.

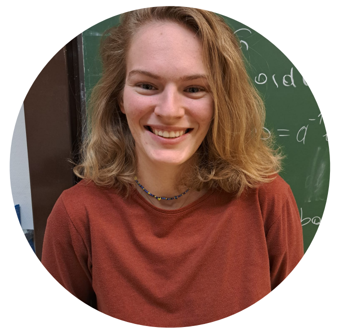
$$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$$

STRATEGIES IN REPEATED GAMES

- Algorithm to verify whether a given reactive-n strategy is an equilibrium.
- It's not just that having more memory gains you nothing. You also gain nothing from having more information.
- Fully characterize cooperative & defective equilibria for $n = 2$ and $n = 3$.
- Under the correct conditions you can have less information.

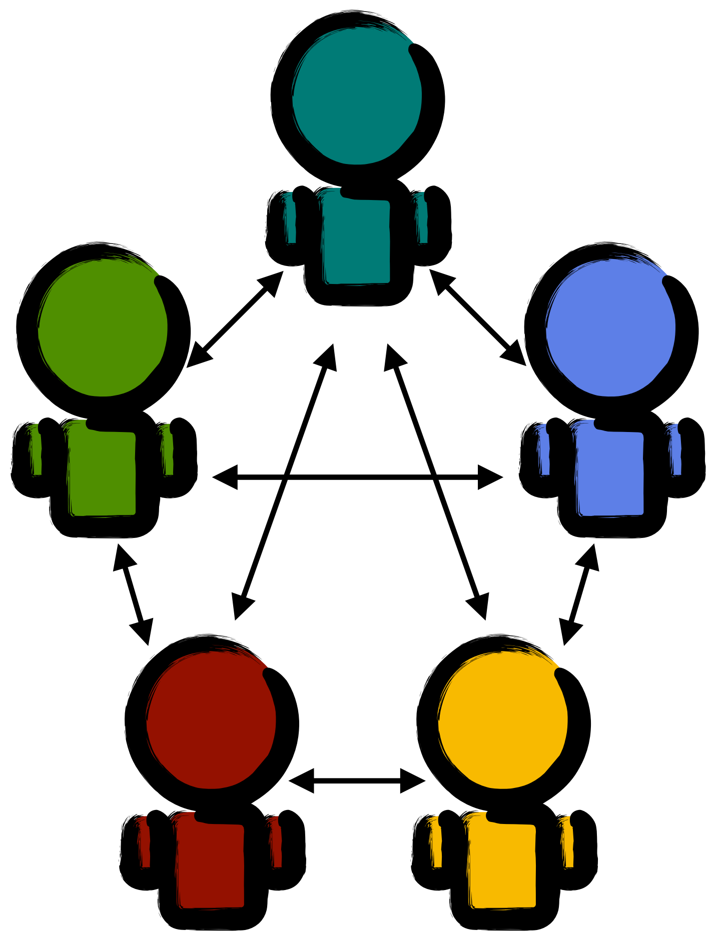
[5] Conditional cooperation with longer
https://doi.org/10.1073/pnas.2420125121

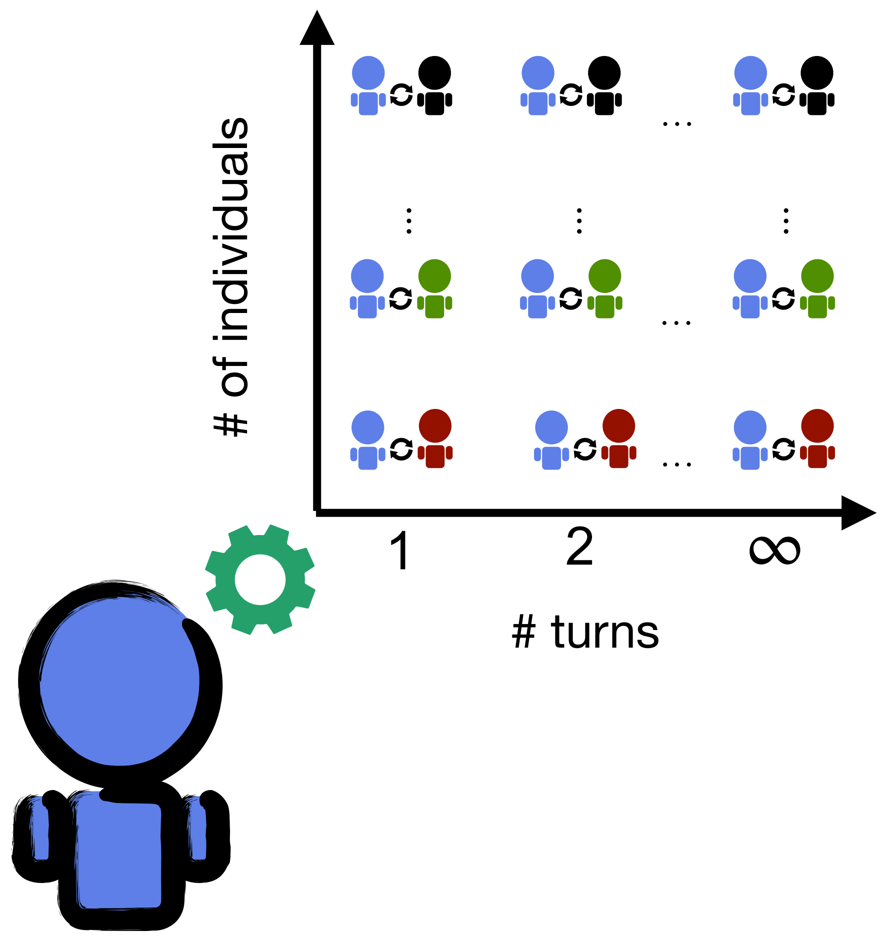
[6] Can I afford to remember less than you?
https://doi.org/10.1016/j.econlet.2025.112300

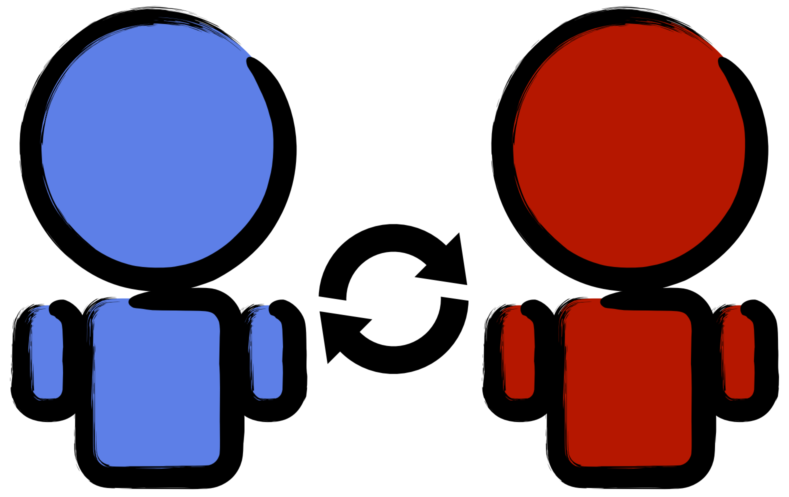


SUMMARY

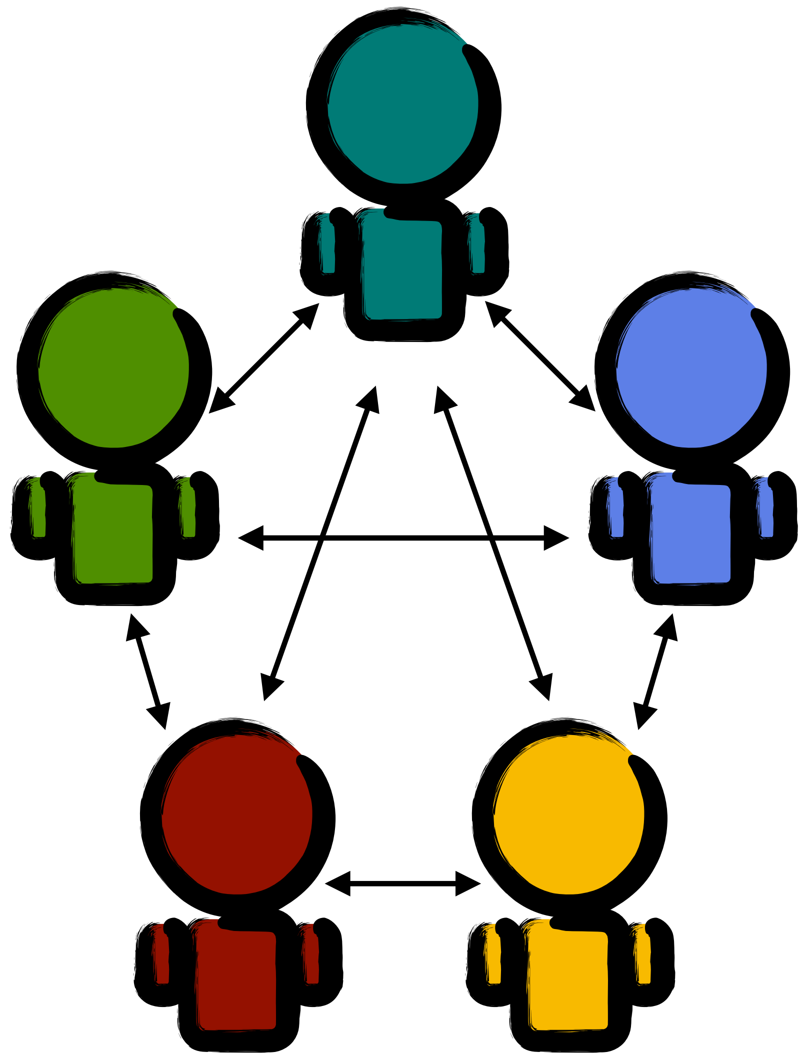
- Current models on direct reciprocity make strong assumptions. Can we explore their impact?
- What kinds of cognitive capacities are required for reciprocal altruism?





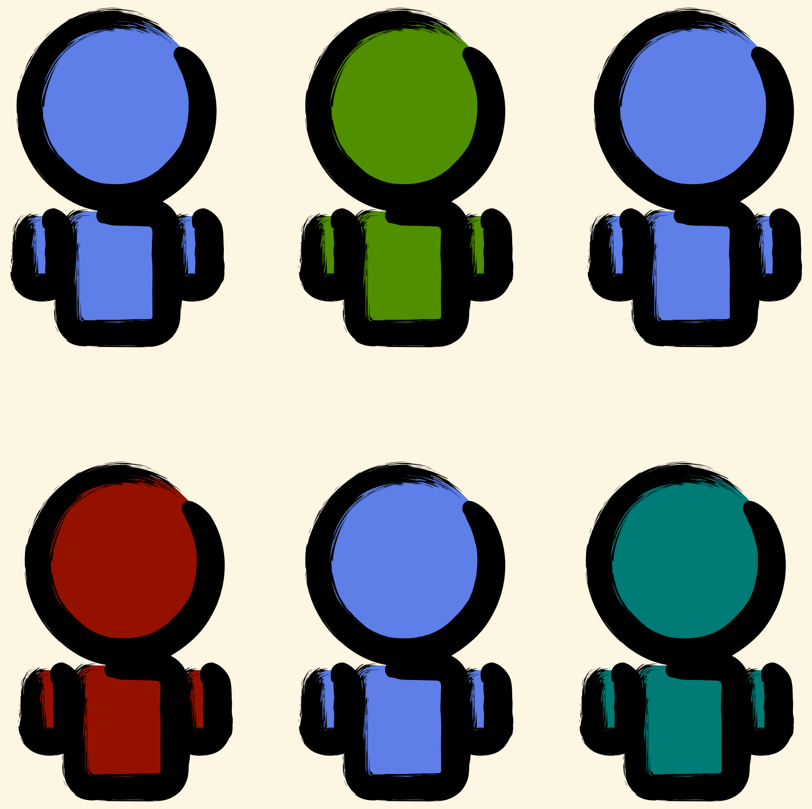


Strategies in computer tournaments



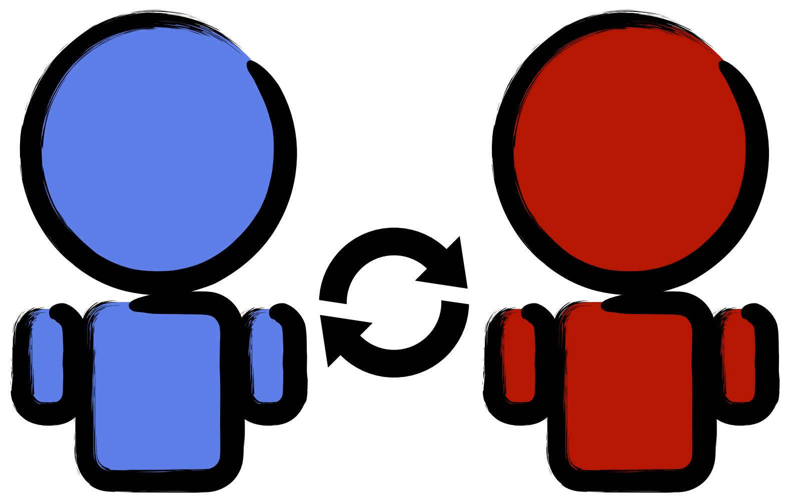
Vincent Knight | Marc Harper | Martin Jones | George Koutsovoulos | Owen Campbell

Learning in populations

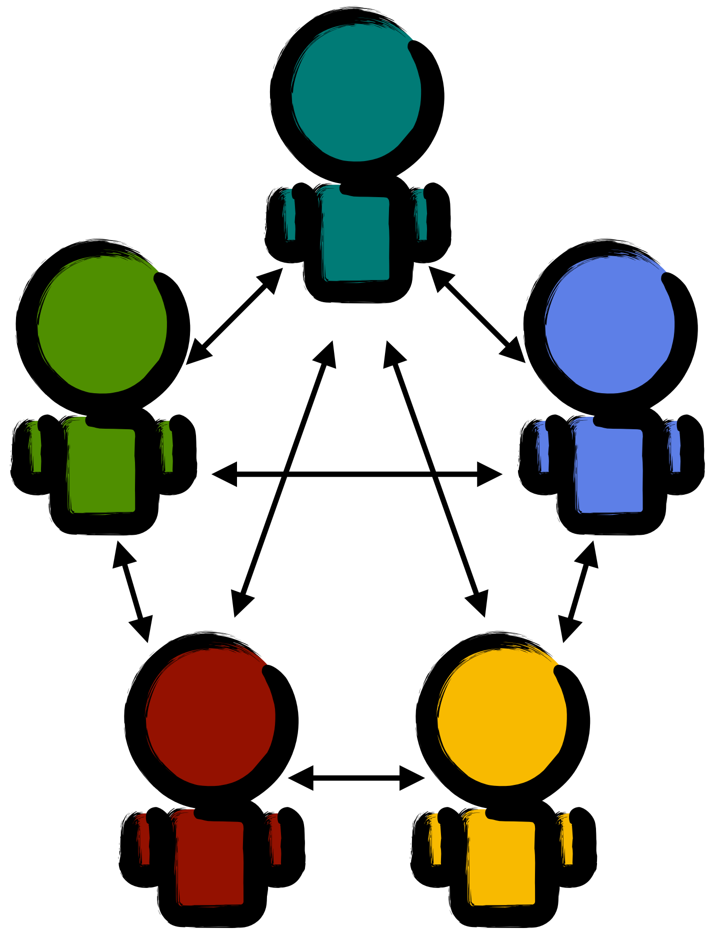


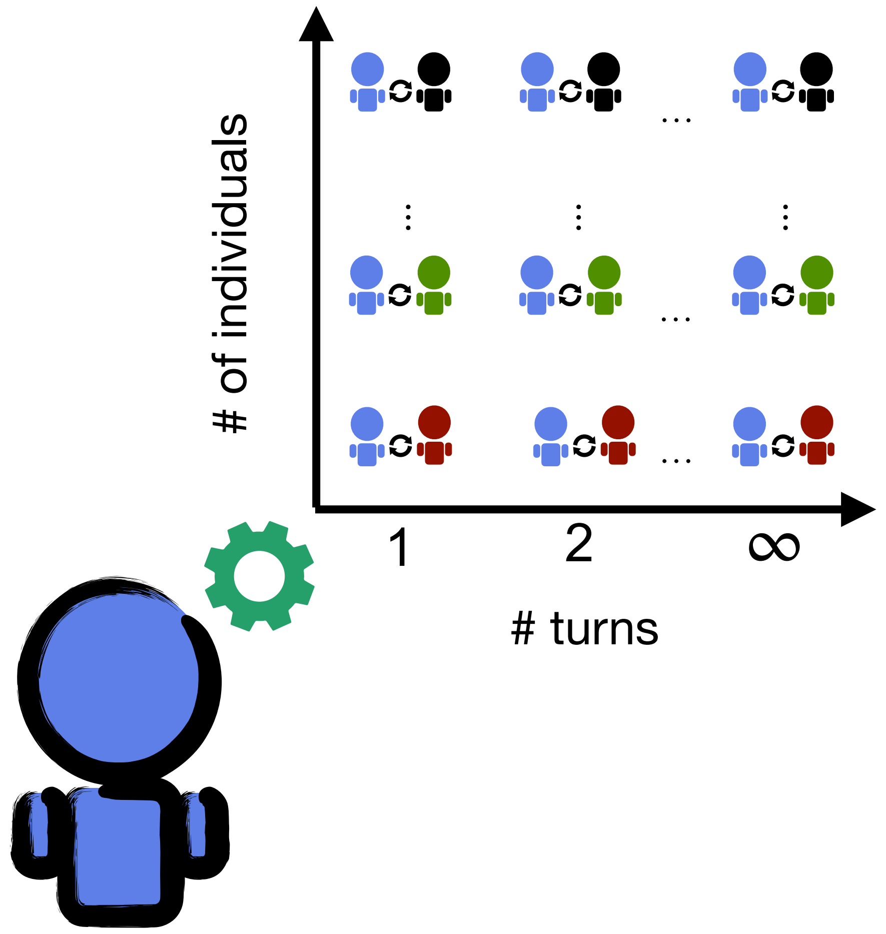
Christian Hilbe | Alex McAvoy

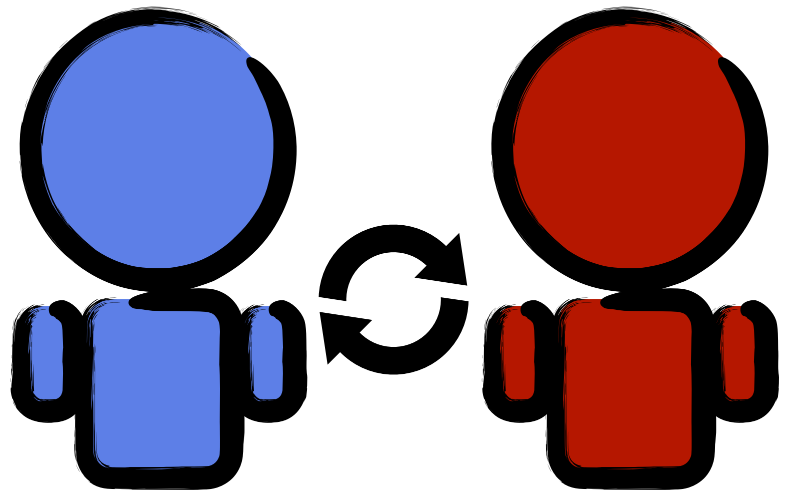
Strategies in repeated interactions



Christian Hilbe | Ethan Akin | Martin Nowak | Franziska Lesigang







Publications

[1] Reinforcement learning produces dominant strategies for the iterated prisoner's dilemma.
https://doi.org/10.1371/journal.pone.0188046

[2] Evolution reinforces cooperation with the emergence of self-recognition mechanisms.
https://doi.org/10.1371/journal.pone.0204981

[3] Properties of winning iterated prisoner's dilemma strategies.
https://doi.org/10.1371/journal.pcbi.1012644

[4] Evolution of reciprocity with limited payoff memory.
https://doi.org/10.1098/rspb.2023.2493

[5] Conditional cooperation with longer.
https://doi.org/10.1073/pnas.2420125121

[6] Can I afford to remember less than you?
https://doi.org/10.1016/j.econlet.2025.112300

Collaborators

Christian Hilbe | Vincent Knight | Marc Harper | Martin Jones | Alex McAvoy | George Koutsovoulos | Owen Campbell | Ethan Akin | Martin Nowak | Franziska Lesigang

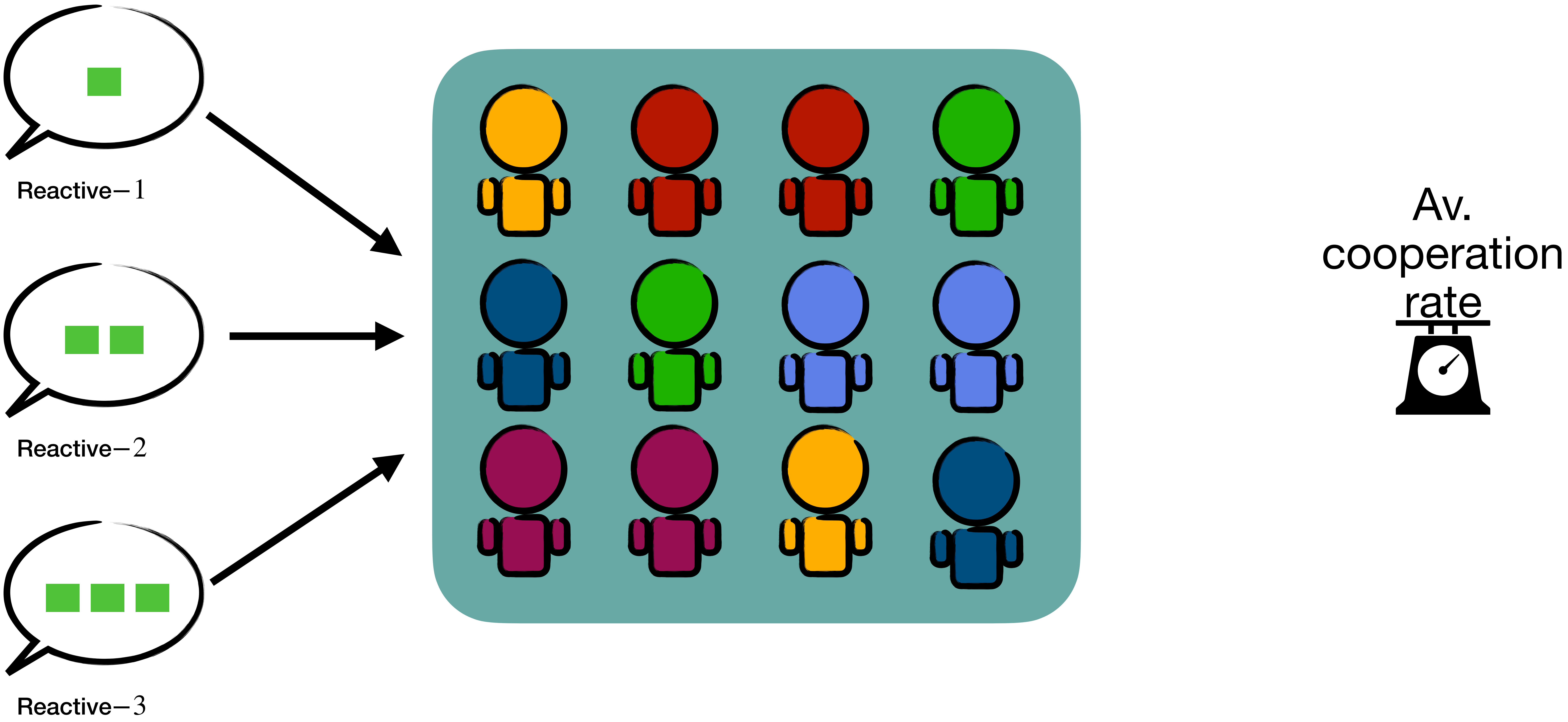
Nikoleta-v3

@NikoletaGlyn

http://nikoleta-v3.github.io

THANK YOU!

STRATEGIES IN REPEATED GAMES



Evolutionary Simulations