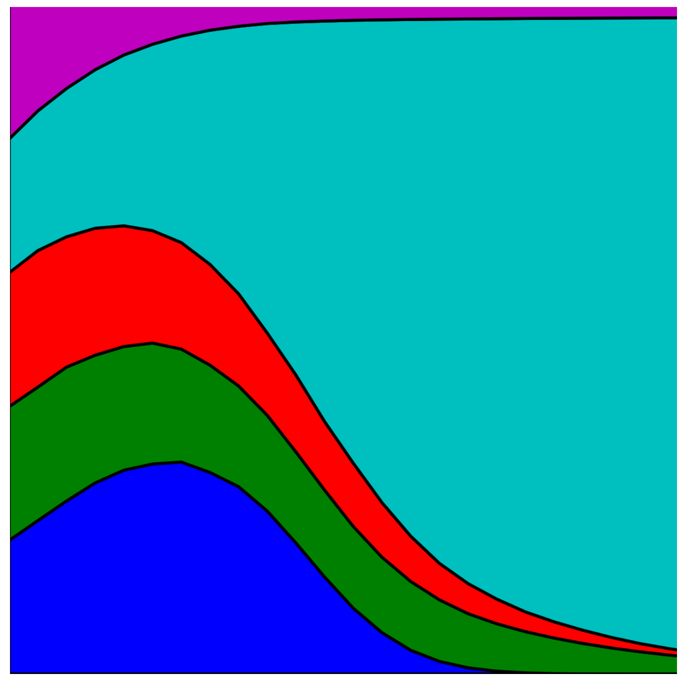
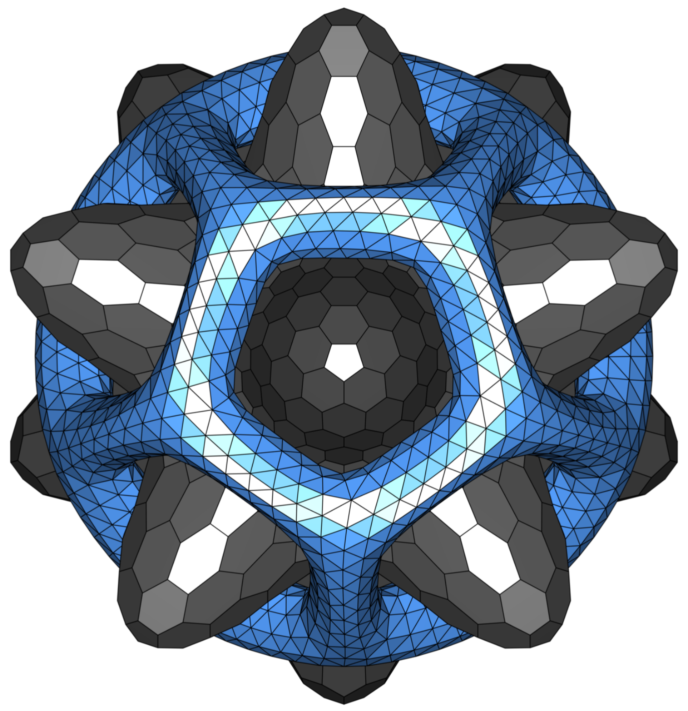
Exploring Cognitive Constraints in Models of Direct Reciprocity

@nikoletaglyn

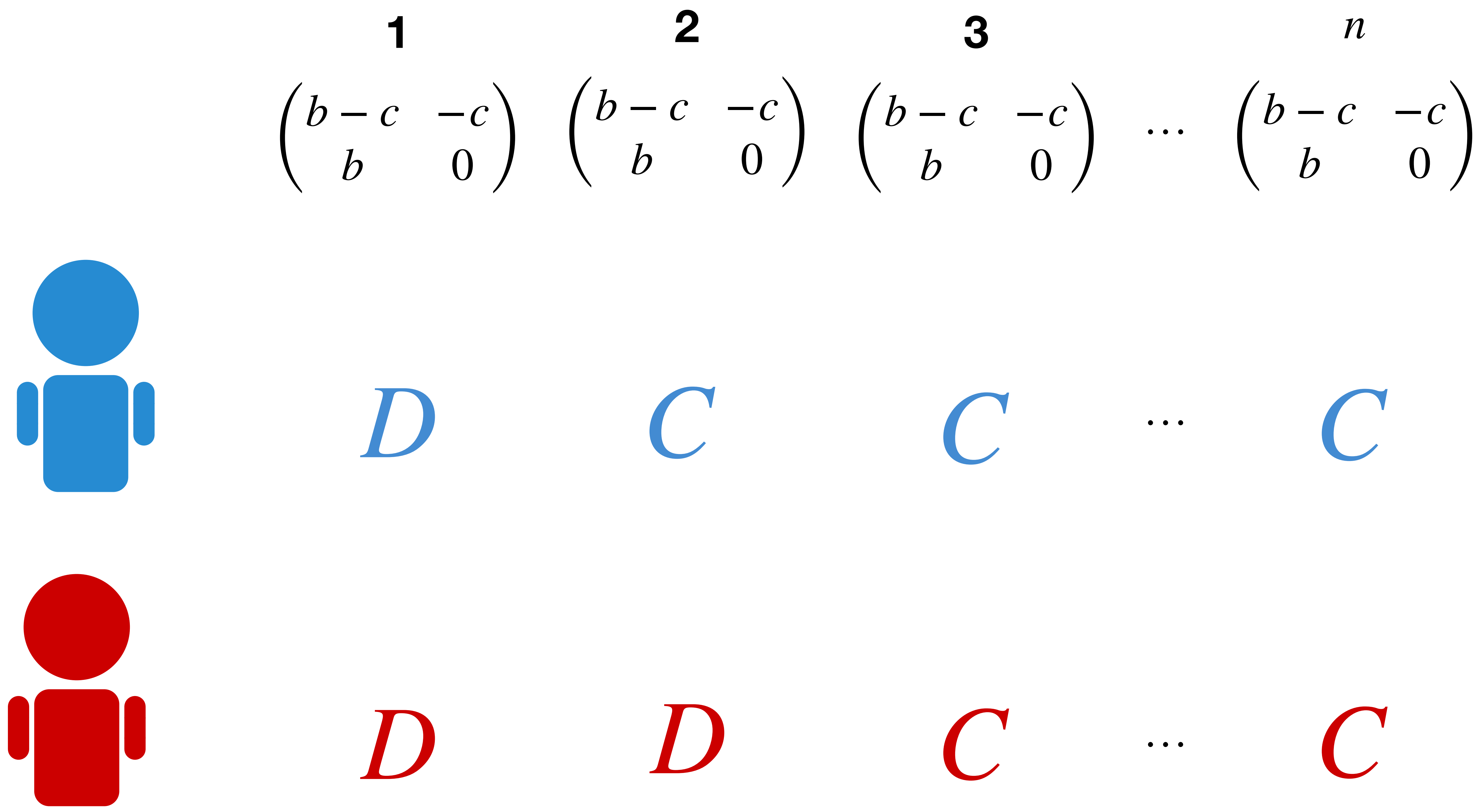
DYNOSOB

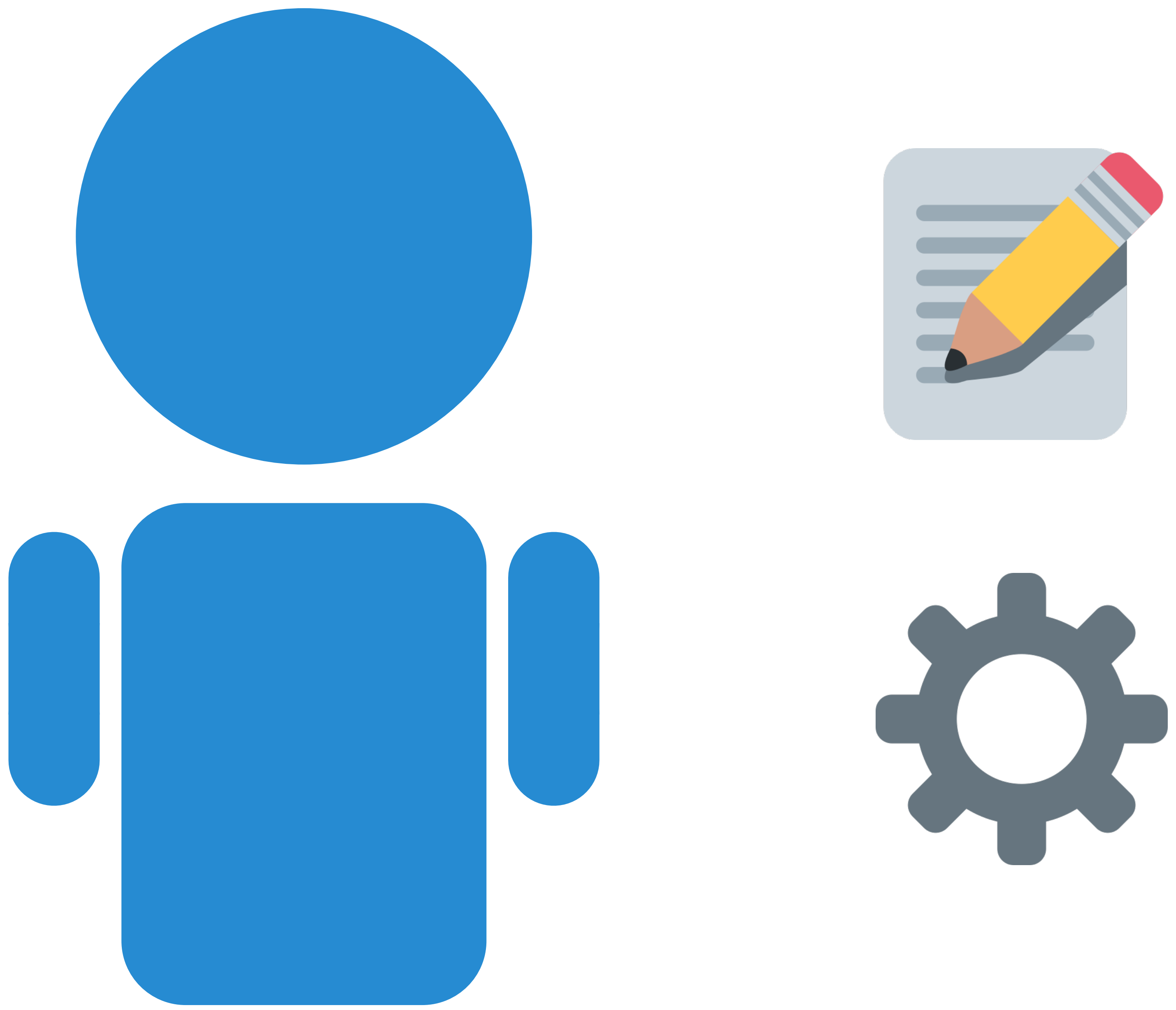


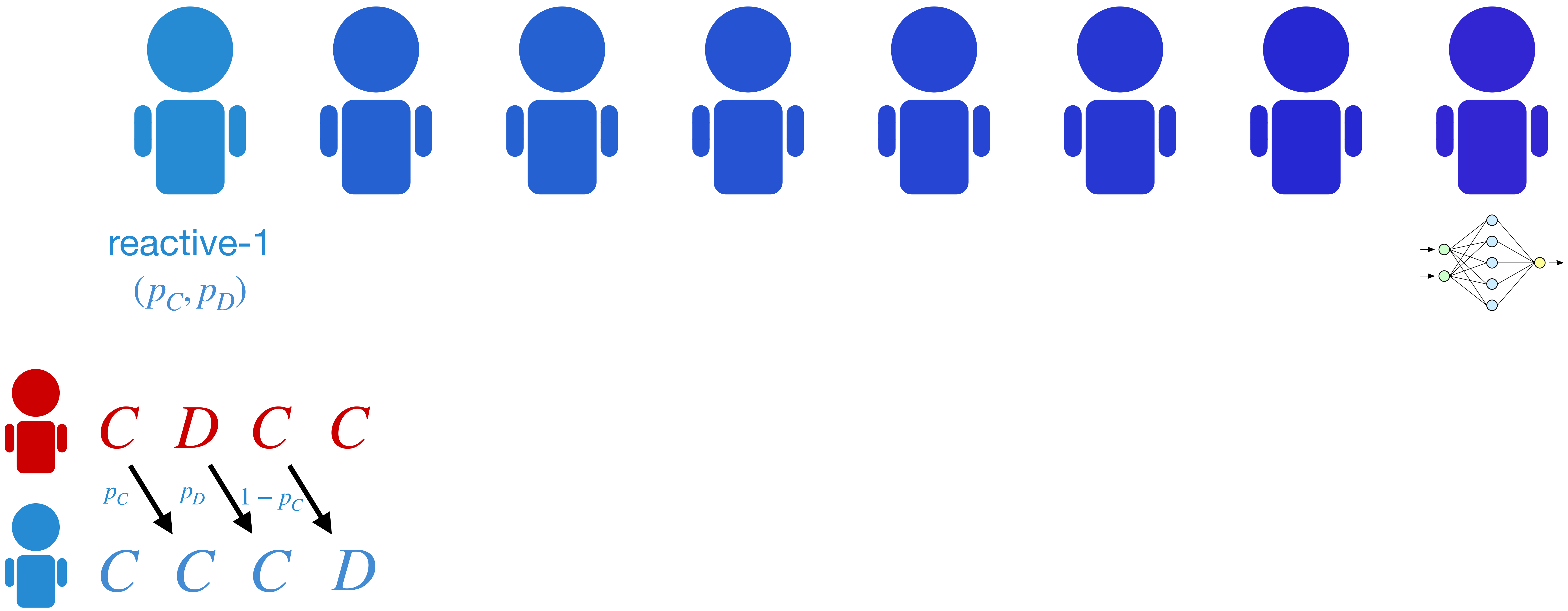


Direct reciprocity is a mechanism for the **emergence of cooperation** in repeated social interactions.

$$
\begin{array}{c} \\ C \\ D \end{array}
\begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array}
$$


1
2
3
n
$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$ $\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$ $\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$ … $\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$
D C C … C
D D C … C




reactive-1
(p_C, p_D)
C D C C
p_C p_D $1-p_C$
C C C D

1. Evolution of reciprocity with limited payoff memory

Alex McAvoy, Christian Hilbe

2. Reactive strategies with longer memory

Martin Nowak, Christian Hilbe

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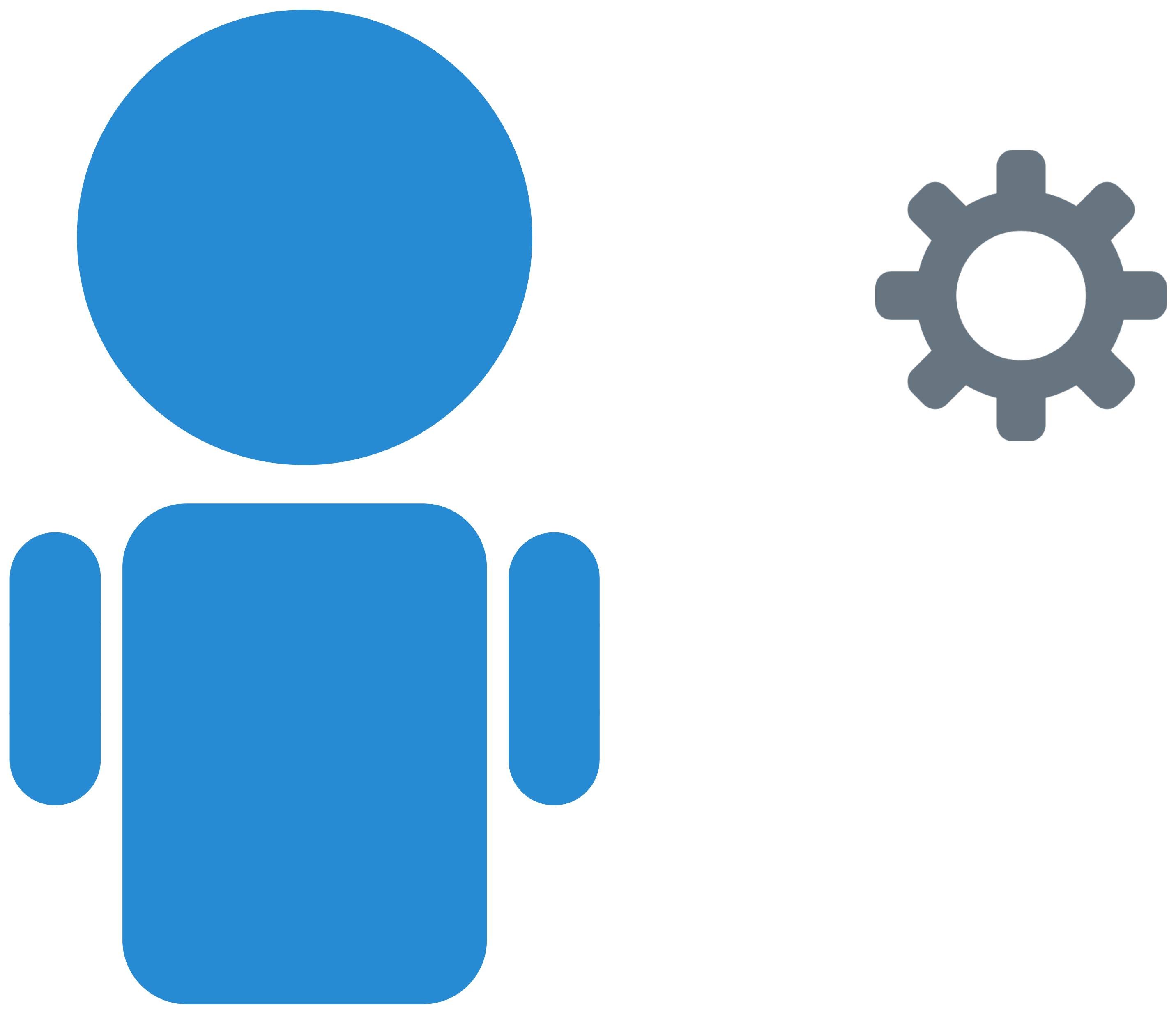
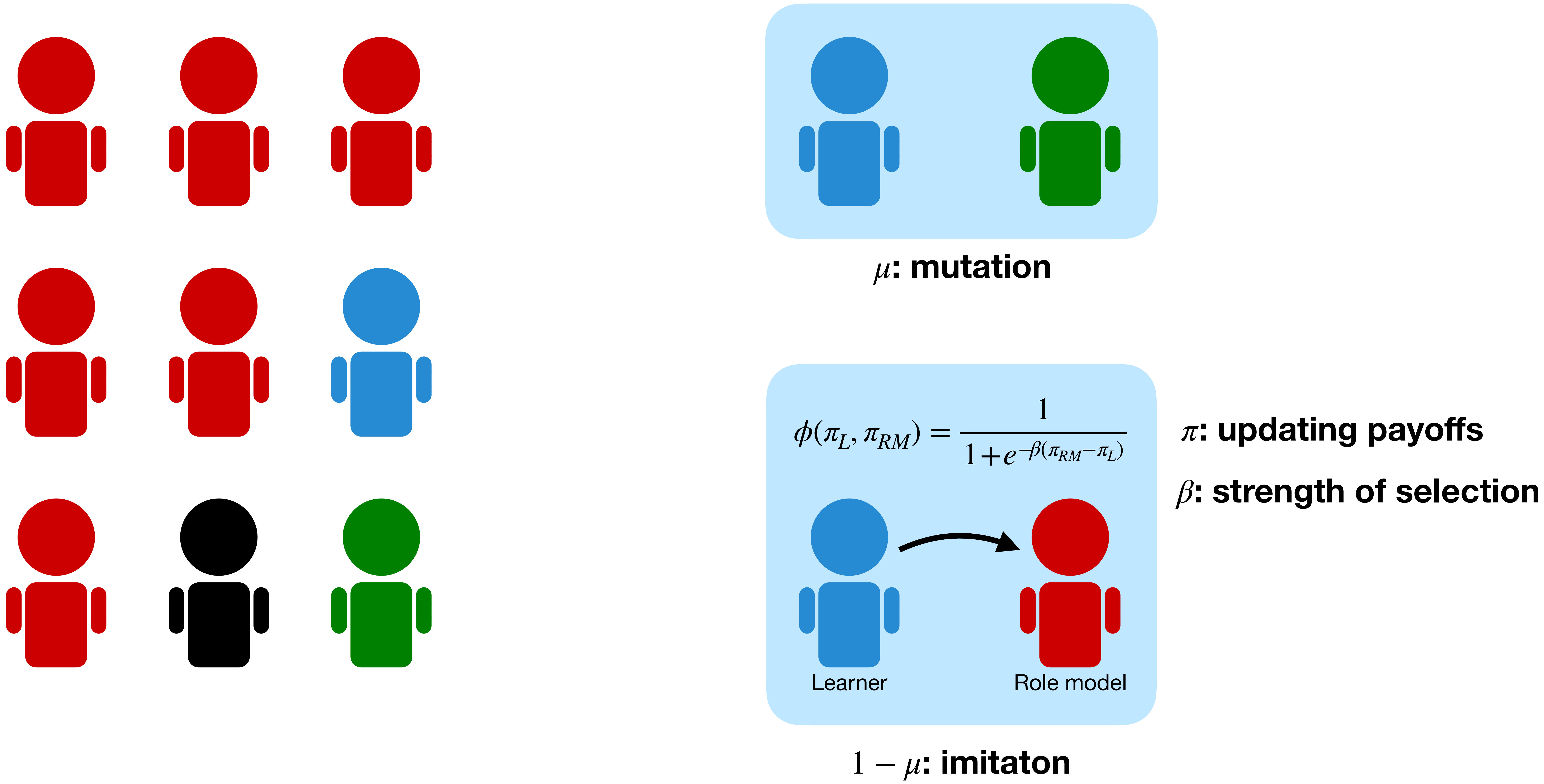
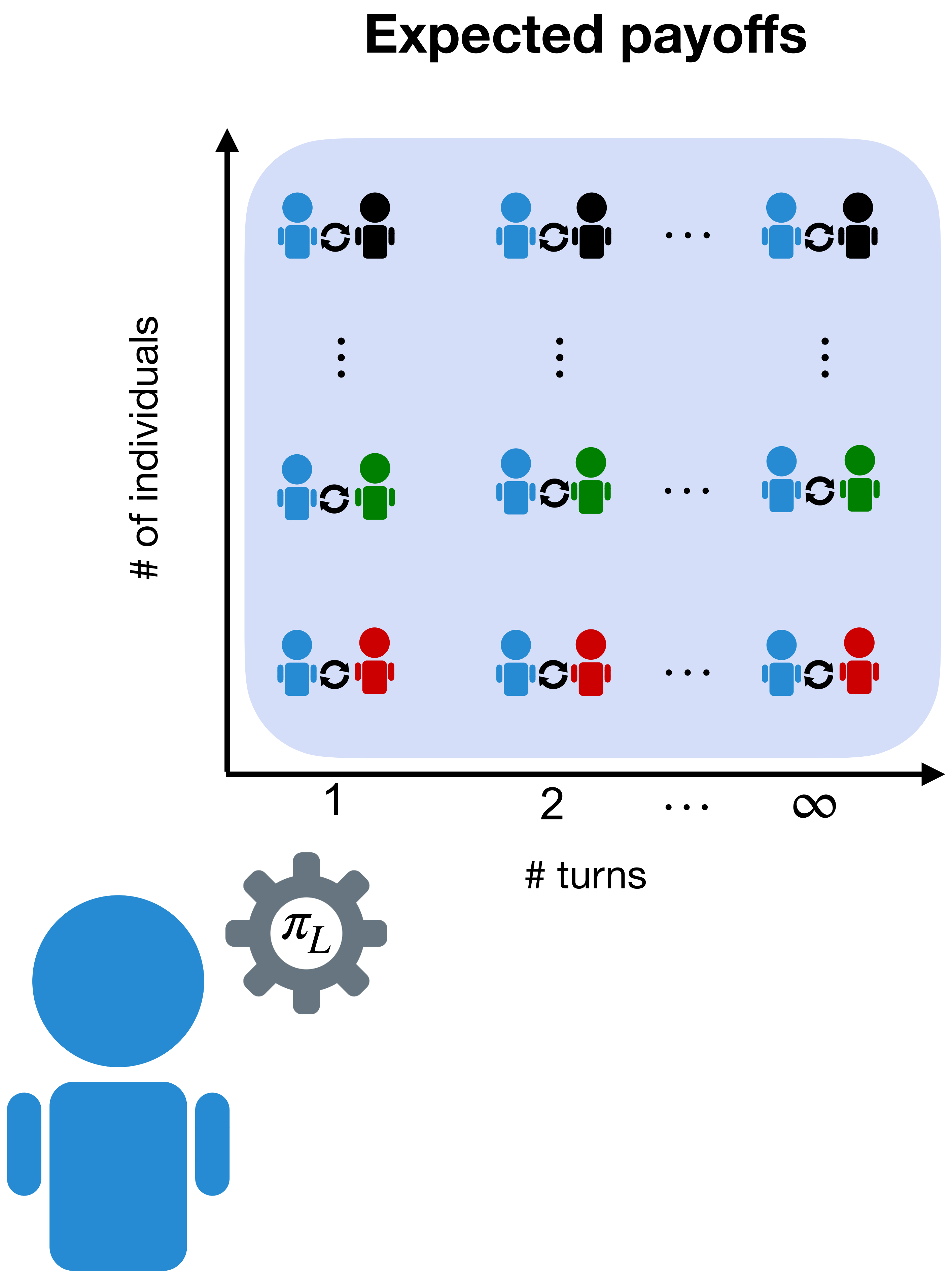
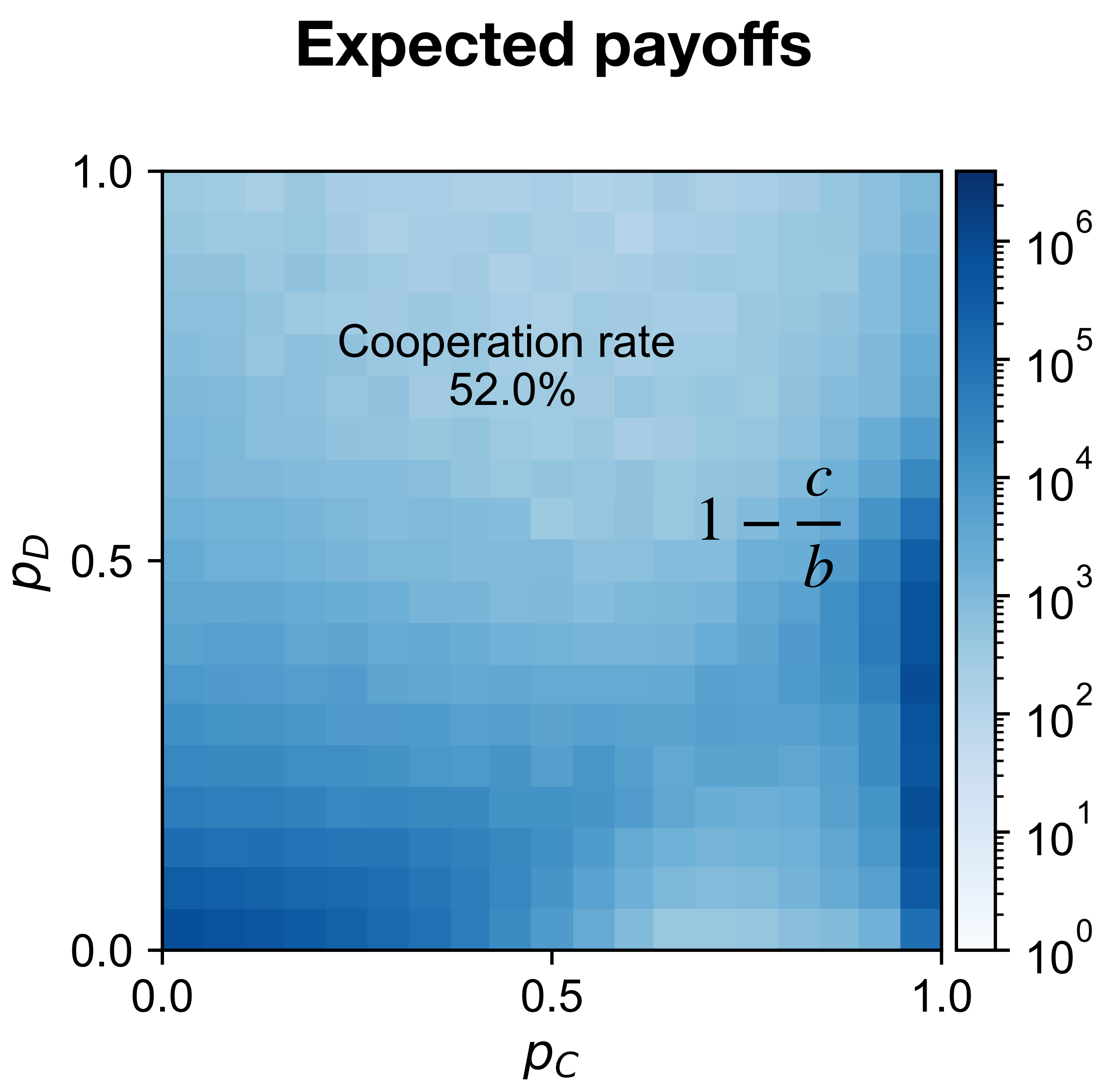



μ: mutation
$\phi(\pi_L, \pi_{RM}) = \frac{1}{1+e^{-\beta(\pi_{RM}-\pi_L)}}$
π: updating payoffs
β: strength of selection
Learner
Role model
$1-\mu$: imitaton

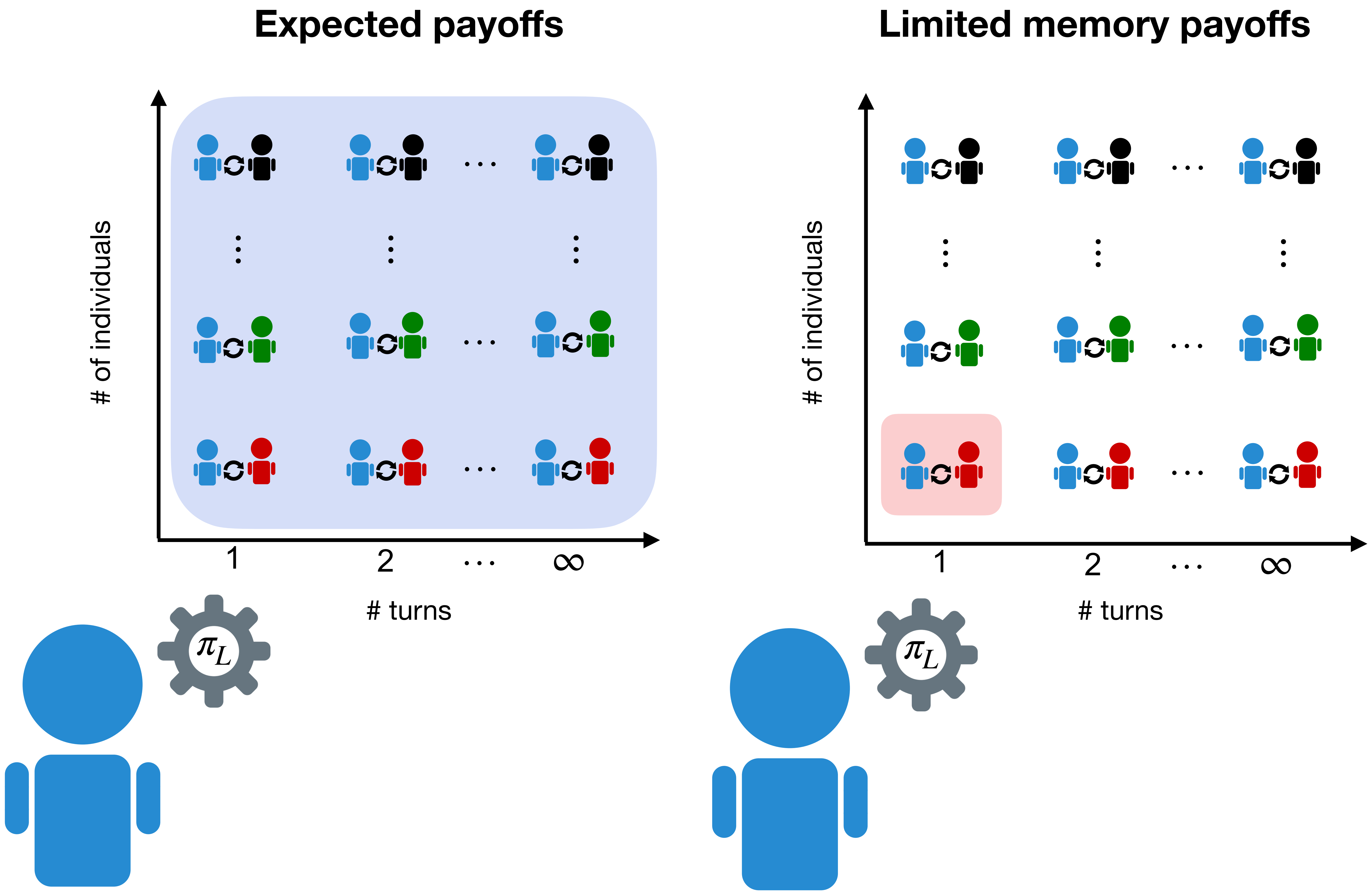


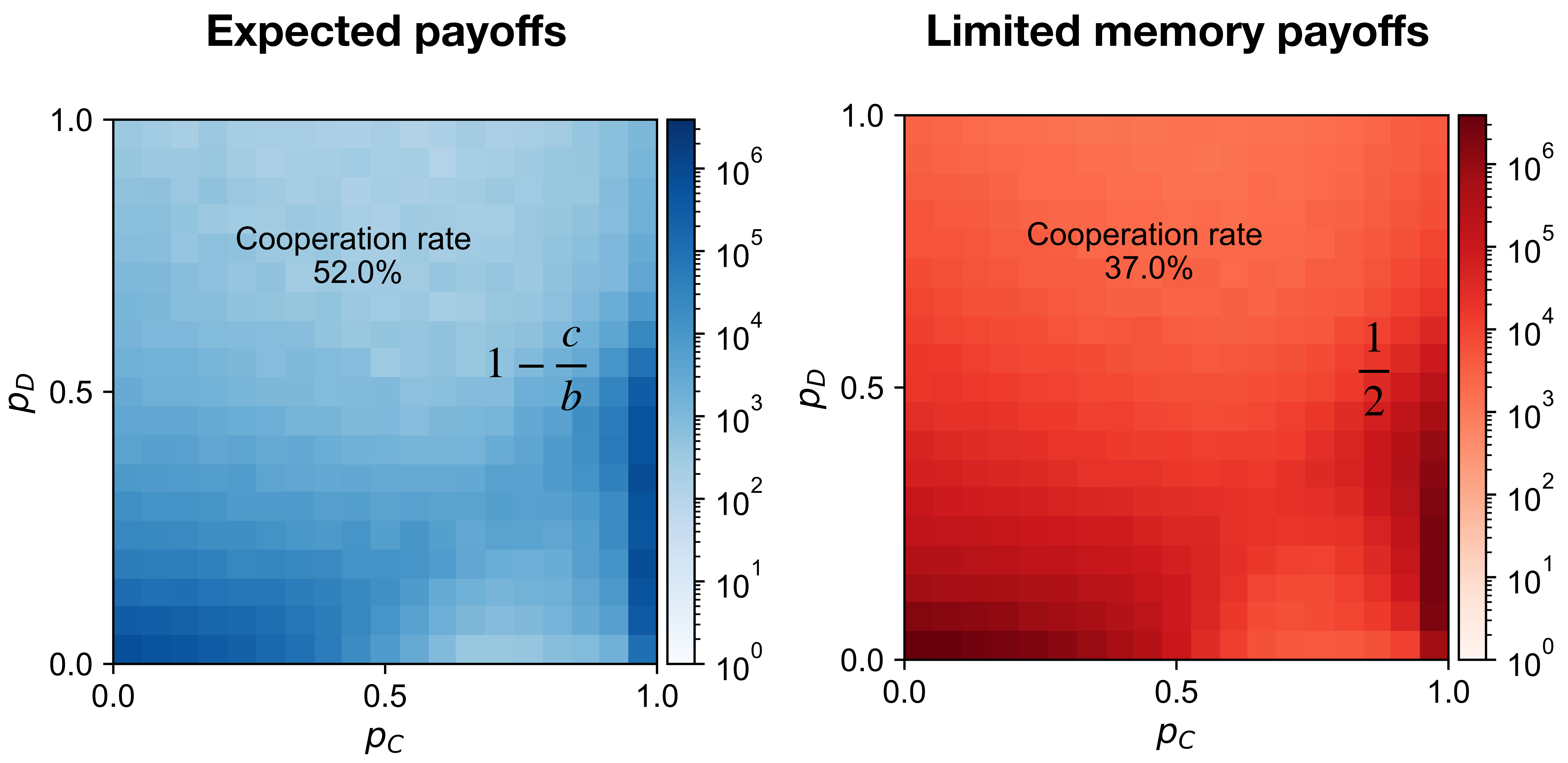
Expected payoffs
# of individuals
1
2
...
∞
turns
π_L



$b = 3$ **and** $c = 1$

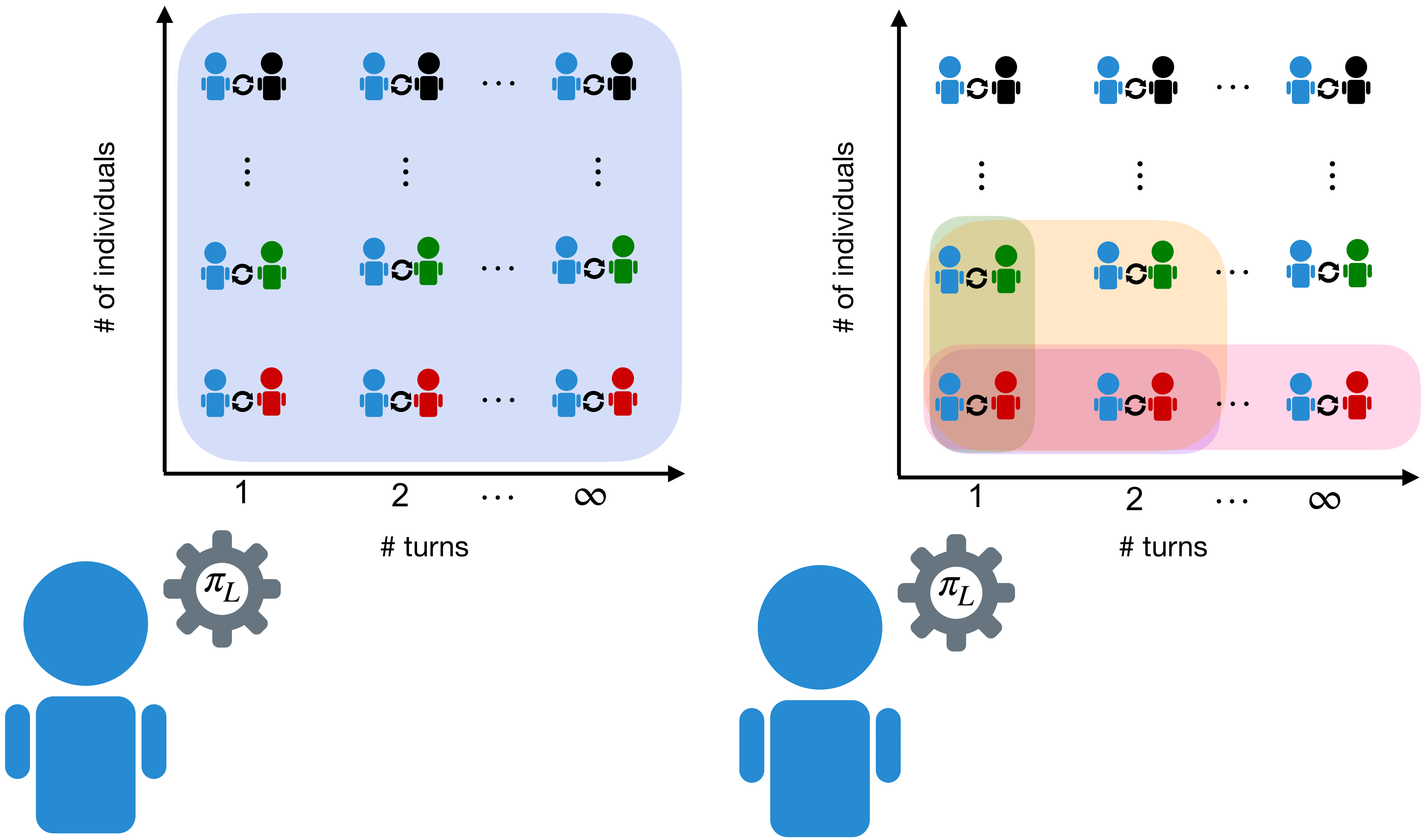
Low mutation $\mu \to 0$


Expected payoffs
Limited memory payoffs
# of individuals
turns
1
2
∞
πL

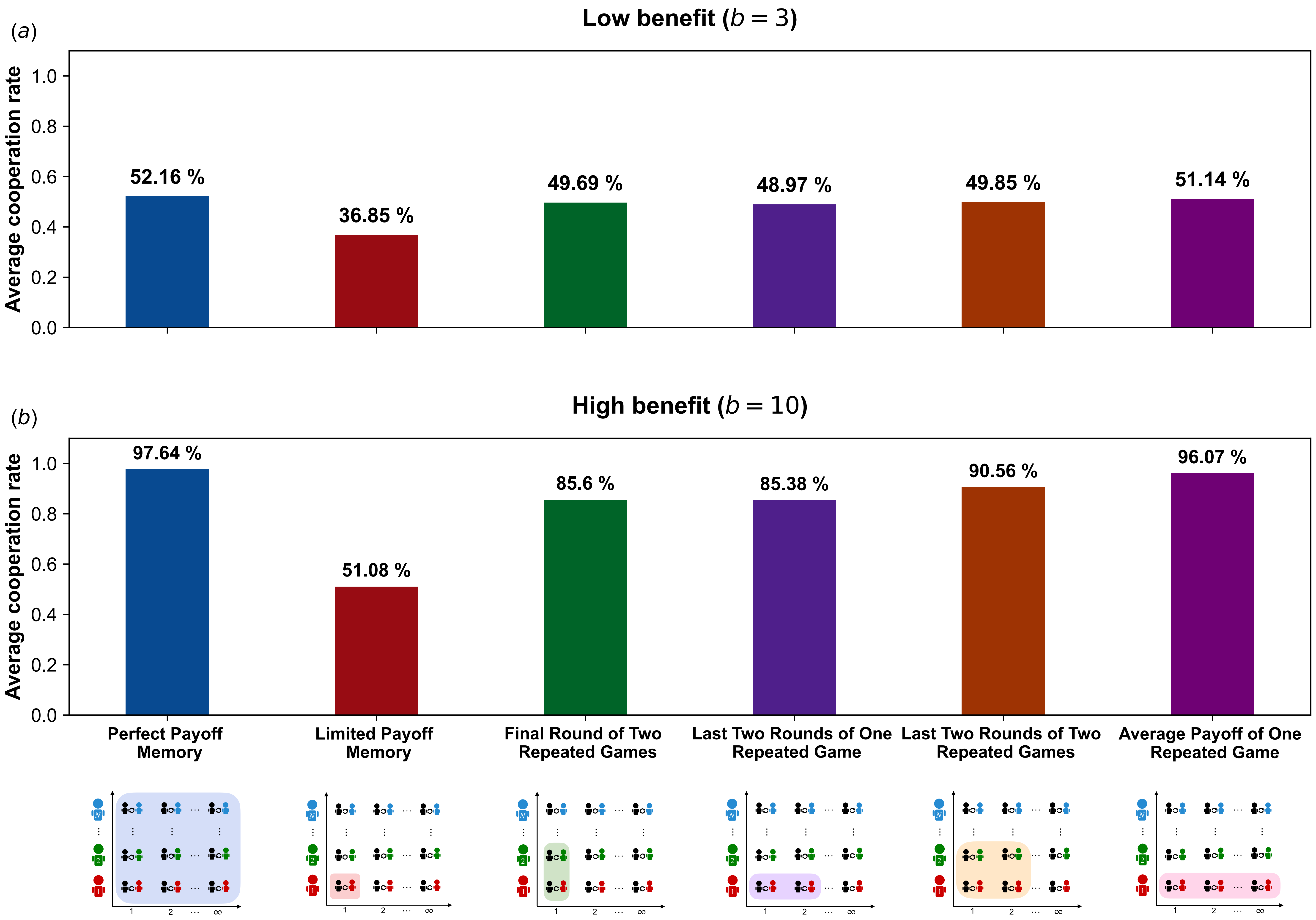


$b = 3$ **and** $c = 1$

Low mutation $\mu \to 0$



# of individuals
1
2
∞
turns
π_L
# of individuals
1
2
∞
turns
π_L


(a)
Low benefit (b = 3)
Average cooperation rate
1.0
0.8
0.6
0.4
0.2
0.0
52.16 %
36.85 %
49.69 %
48.97 %
49.85 %
51.14 %
(b)
High benefit (b = 10)
Average cooperation rate
97.64 %
51.08 %
85.6 %
85.38 %
90.56 %
96.07 %
Perfect Payoff Memory
Limited Payoff Memory
Final Round of Two Repeated Games
Last Two Rounds of One Repeated Game
Last Two Rounds of Two Repeated Games
Average Payoff of One Repeated Game

https://arxiv.org/abs/2311.02365

- Even with individuals considering only their last interaction, cooperation can still evolve.

- However, strategies tend to be less generous and cooperate less frequently.

- As individuals recall the payoffs of two or three recent interactions, the cooperation rates approach the classical limit.

1. Evolution of reciprocity with limited payoff memory

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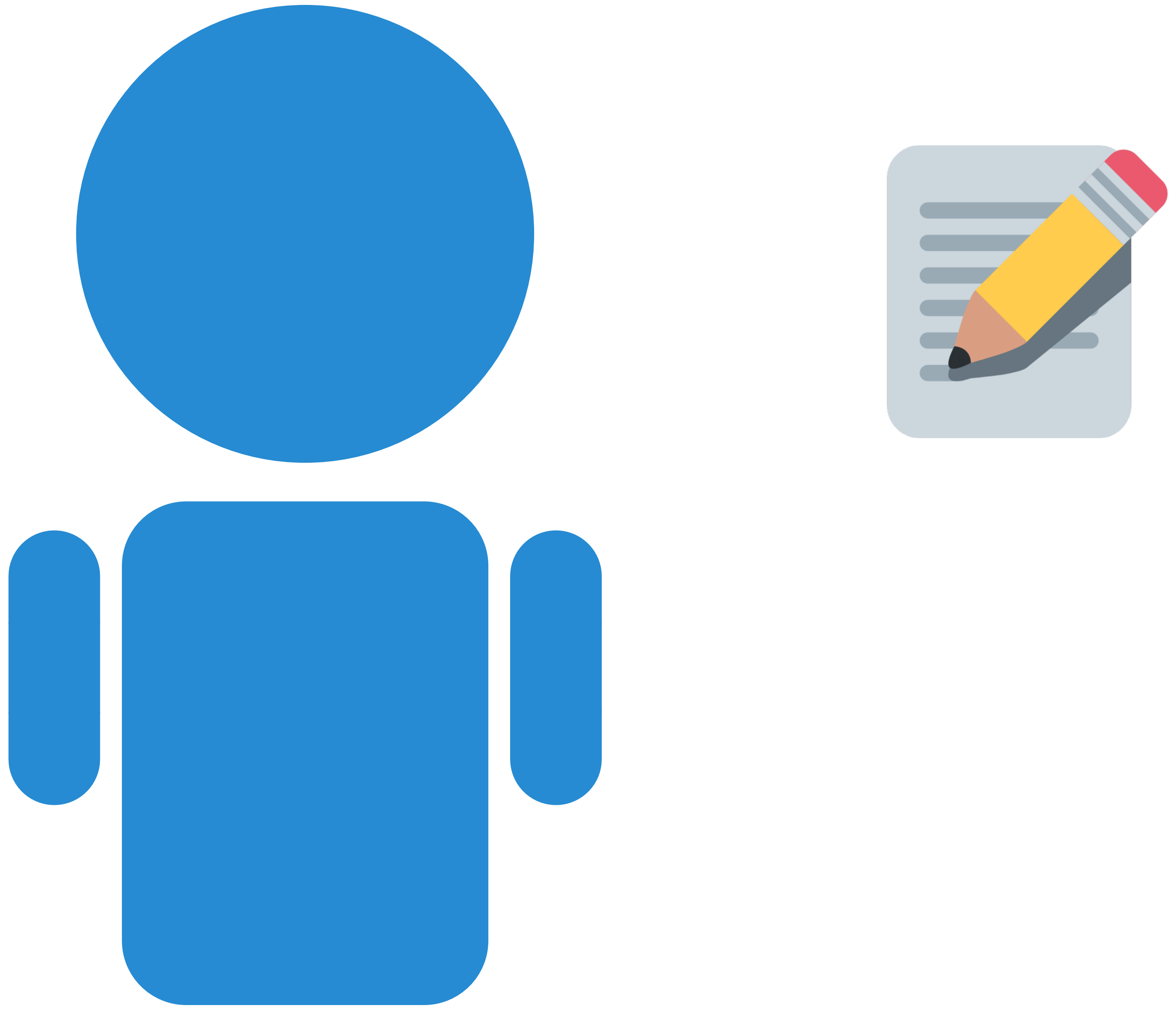
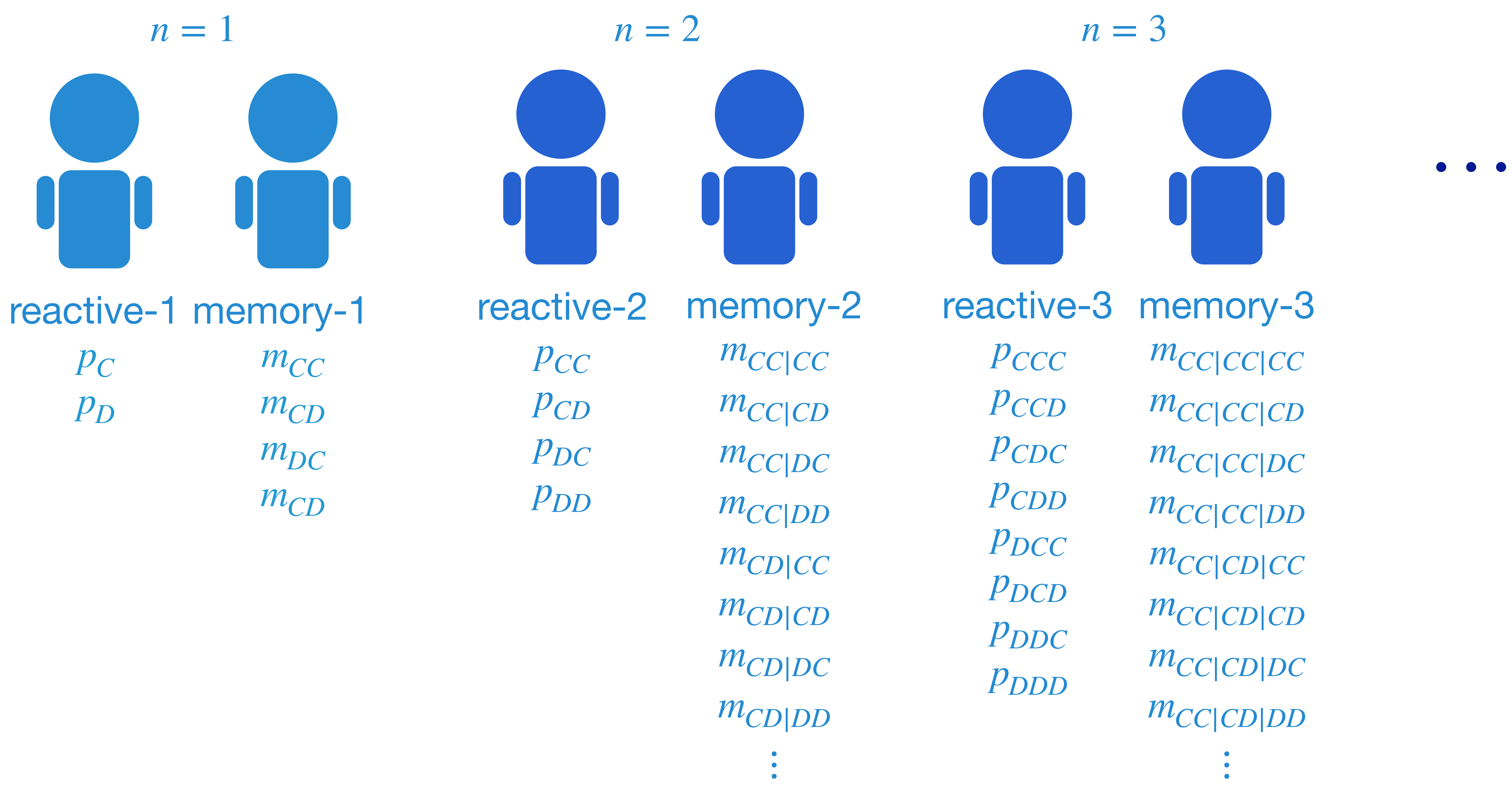
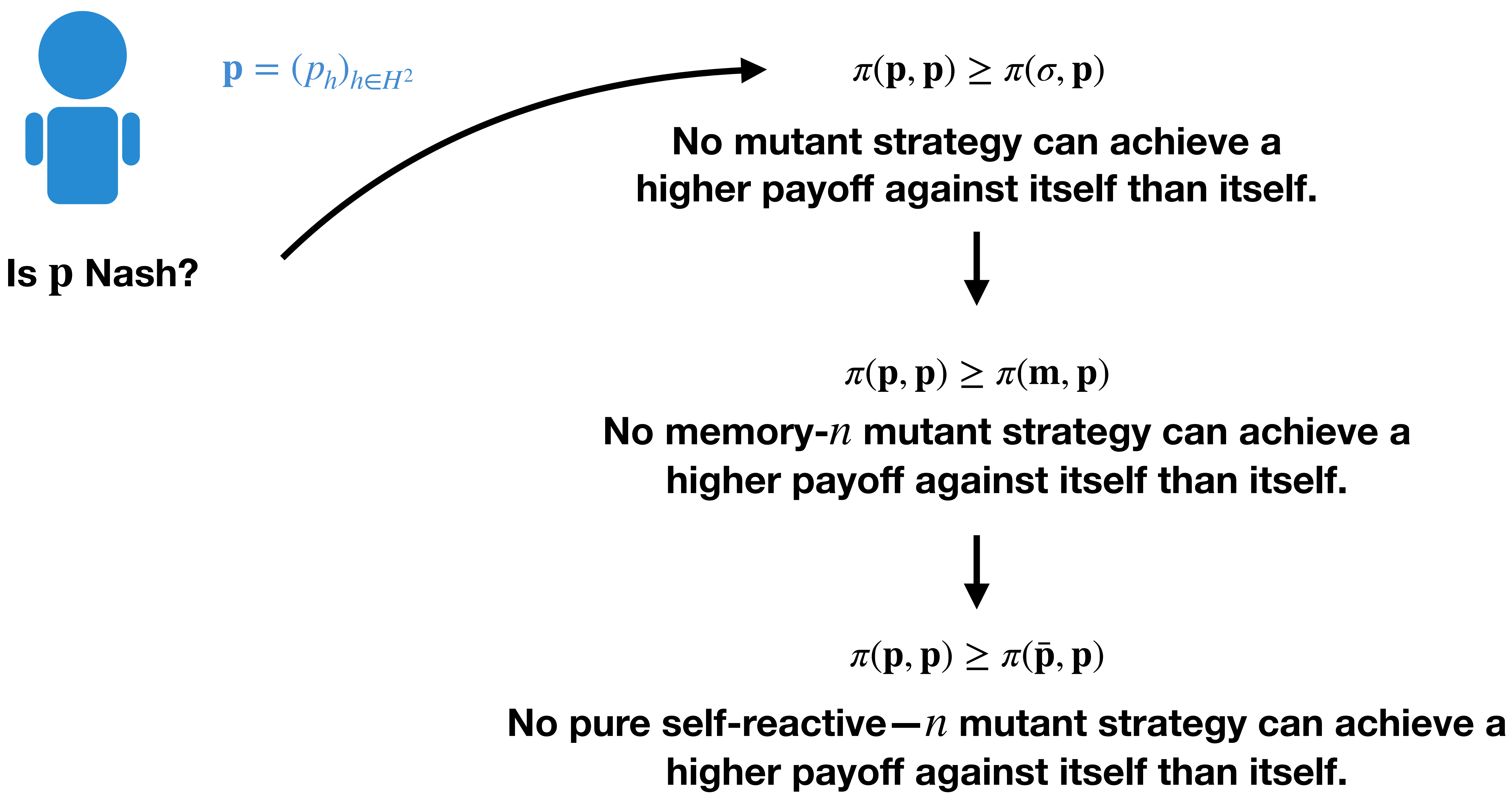
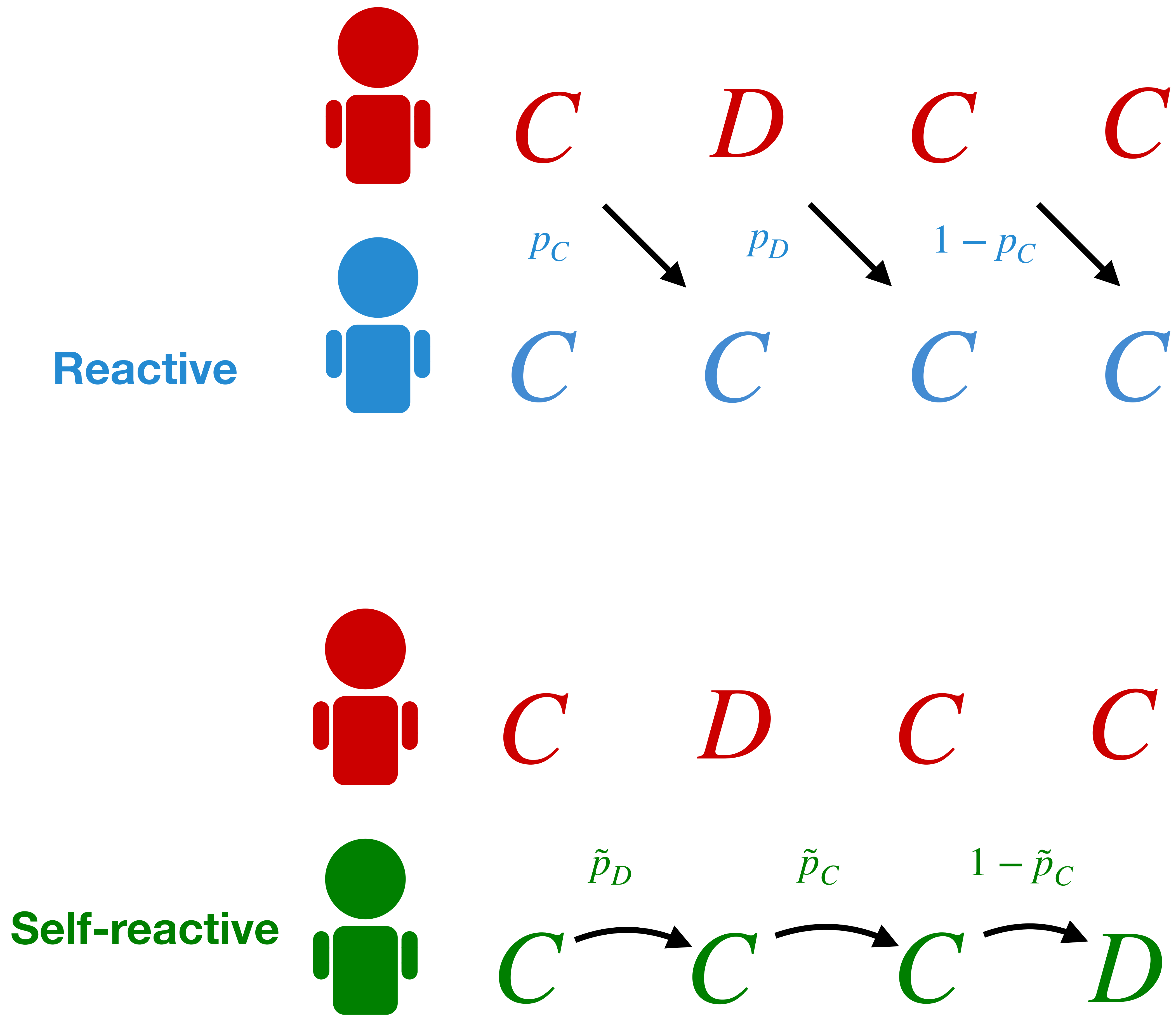




n = 1
n = 2
n = 3
reactive-1
memory-1
reactive-2
memory-2
reactive-3
memory-3
p_C
p_D
m_{CC}
m_{CD}
m_{DC}
m_{CD}
p_{CC}
p_{CD}
p_{DC}
p_{DD}
$m_{CC|CC}$
$m_{CC|CD}$
$m_{CC|DC}$
$m_{CC|DD}$
$m_{CD|CC}$
$m_{CD|CD}$
$m_{CD|DC}$
$m_{CD|DD}$
p_{CCC}
p_{CCD}
p_{CDC}
p_{CDD}
p_{DCC}
p_{DCD}
p_{DDC}
p_{DDD}
$m_{CC|CC|CC}$
$m_{CC|CC|CD}$
$m_{CC|CC|DC}$
$m_{CC|CC|DD}$
$m_{CC|CD|CC}$
$m_{CC|CD|CD}$
$m_{CC|CD|DC}$
$m_{CC|CD|DD}$


$\mathbf{p} = (p_h)_{h \in H^2}$
Is p Nash?
$\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\sigma, \mathbf{p})$
No mutant strategy can achieve a higher payoff against itself than itself.
$\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\mathbf{m}, \mathbf{p})$
No memory-n mutant strategy can achieve a higher payoff against itself than itself.
$\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\bar{\mathbf{p}}, \mathbf{p})$
No pure self-reactive—n mutant strategy can achieve a higher payoff against itself than itself.



Reactive
Self-reactive
C D C C
p_C
p_D
$1 - p_C$
C C C C
C D C C
$\tilde{p}_D$
$\tilde{p}_C$
$1 - \tilde{p}_C$
C C C D

A reactive$-n$ strategy $\mathbf{p}$, is a Nash strategy if, and only if, no pure self-reactive$-n$ strategy can achieve a higher payoff against itself.

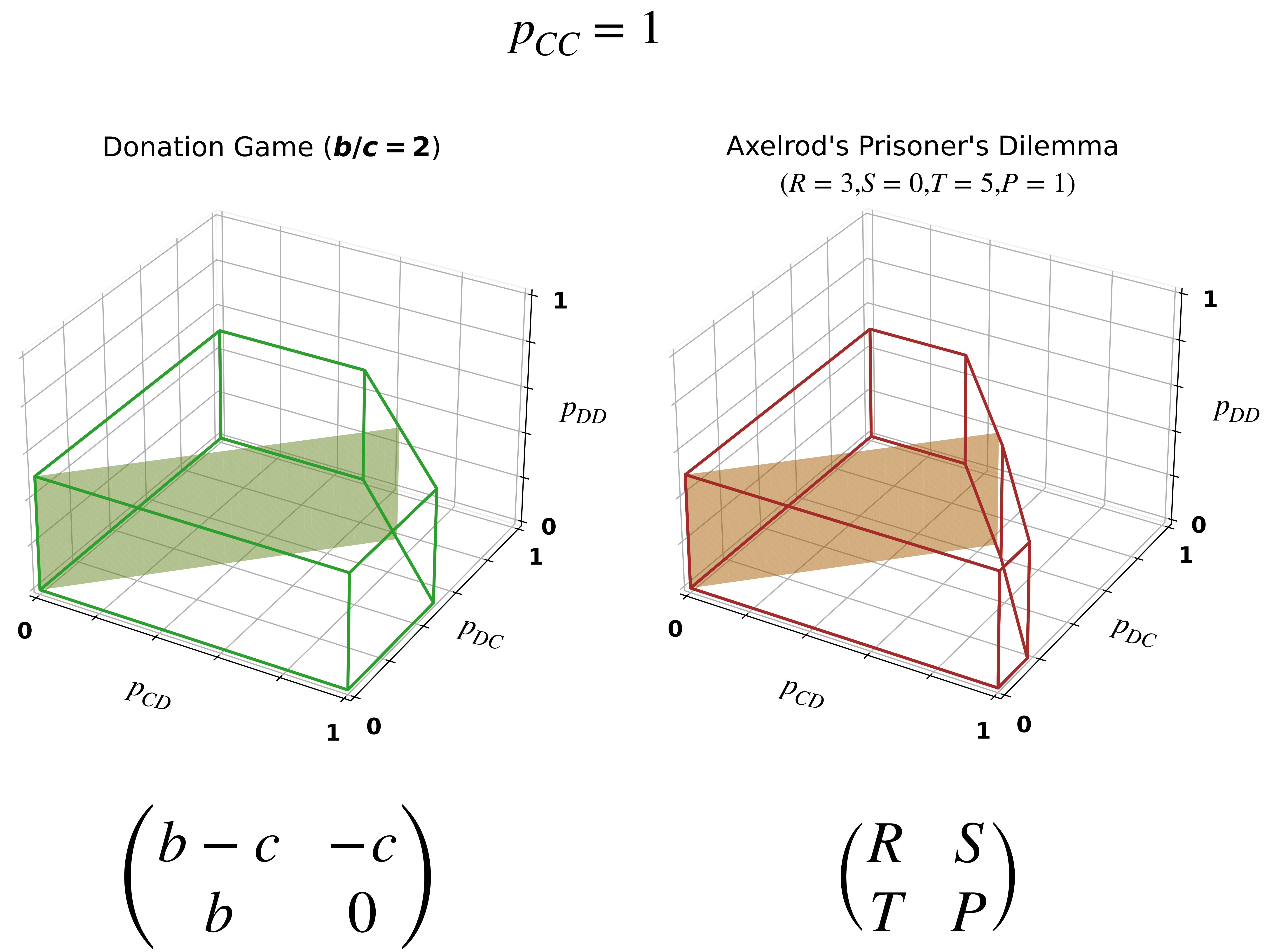
We use this result to characterise cooperative Nash equilibria (partners) among reactive-2 and reactive-3 strategies.

A reactive-2 strategy can be defined as the vector $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$, and it is a cooperative Nash strategy if and only if, the strategy entries satisfy the conditions,

$$p_{CC} = 1, \qquad \frac{p_{CD} + p_{DC}}{2} < 1 - \frac{1}{2} \cdot \frac{c}{b} \qquad \textit{and} \qquad p_{DD} \leq 1 - \frac{c}{b}\,.$$

A reactive-3 strategy is defined by the vector $\mathbf{p} = (p_{CCC}, p_{CCD}, p_{CDC}, p_{CDD}, p_{DCC}, p_{DCD}, p_{DDC}, p_{DDD})$, and it is a cooperative Nash strategy, if and only if the strategy entries satisfy the conditions,

$$\begin{aligned}
p_{CCC} &= 1 \\
\frac{p_{CDC} + p_{DCD}}{2} &\leq 1 - \frac{1}{2} \cdot \frac{c}{b} \\
\frac{p_{CCD} + p_{CDC} + p_{DCC}}{3} &\leq 1 - \frac{1}{3} \cdot \frac{c}{b} \\
\frac{p_{CDD} + p_{DCD} + p_{DDC}}{3} &\leq 1 - \frac{2}{3} \cdot \frac{c}{b} \\
\frac{p_{CCD} + p_{CDD} + p_{DCC} + p_{DDC}}{4} &\leq 1 - \frac{1}{2} \cdot \frac{c}{b} \\
p_{DDD} &\leq 1 - \frac{c}{b}
\end{aligned}$$



$p_{CC} = 1$
Donation Game (b/c = 2)
Axelrod's Prisoner's Dilemma
(R = 3,S = 0,T = 5,P = 1)
p_{DD}
p_{DC}
p_{CD}
0
1
$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$
$\begin{pmatrix} R & S \\ T & P \end{pmatrix}$

On arXiv soon

- A general algorithm to assess whether a given reactive-n strategy is a Nash equilibrium
- Explicitly characterise cooperative Nash equilibria among reactive-2 and reactive-3 strategies

- I wanted to convey that our models rely on assumptions, and it is sometimes beneficial to relax them to better understand their effects.

- We have made progress in analyzing higher-memory strategies for repeated games.

My collaborators

Alex McAvoy & Christian Hilbe & Martin Nowak

Special thank you to Ethan Akin

More information

@nikoletaglyn

http://web.evolbio.mpg.de/social-behaviour/

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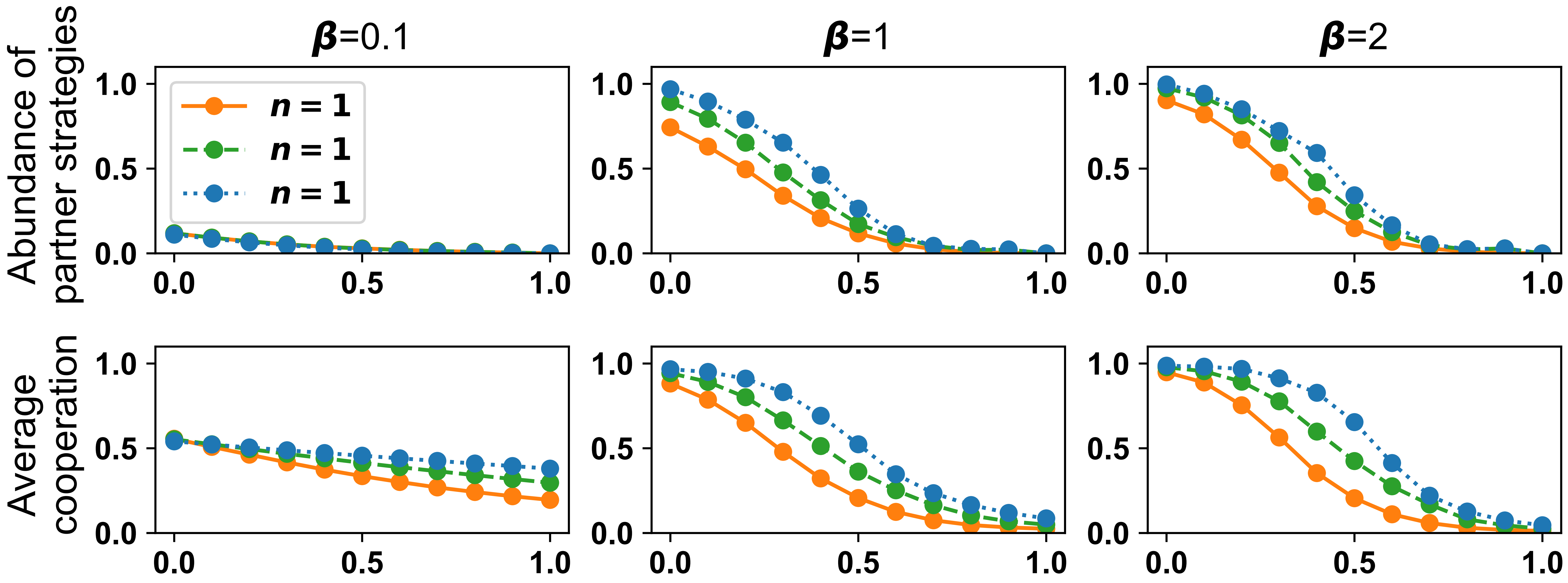

β=0.1
β=1
β=2
Abundance of partner strategies
Average cooperation
n = 1
n = 1
n = 1
1.0
0.5
0.0
0.0
0.5
1.0